

Power Screws

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17.1 Introduction

The power screws (also known as *translation screws*) are used to convert rotary motion into translatory motion. For example, in the case of the lead screw of lathe, the rotary motion is available but the tool has to be advanced in the direction of the cut against the cutting resistance of the material. In case of screw jack, a small force applied in the horizontal plane is used to raise or lower a large load. Power screws are also used in vices, testing machines, presses, etc.

In most of the power screws, the nut has axial motion against the resisting axial force while the screw rotates in its bearings. In some screws, the screw rotates and moves axially against the resisting force while the nut is stationary and in others the nut rotates while the screw moves axially with no rotation.

17.2 Types of Screw Threads used for Power Screws

Following are the three types of screw threads mostly used for power screws :

1. Square thread. A square thread, as shown in Fig. 17.1 (a), is adapted for the transmission of power in either direction. This thread results in maximum efficiency and minimum radial or bursting

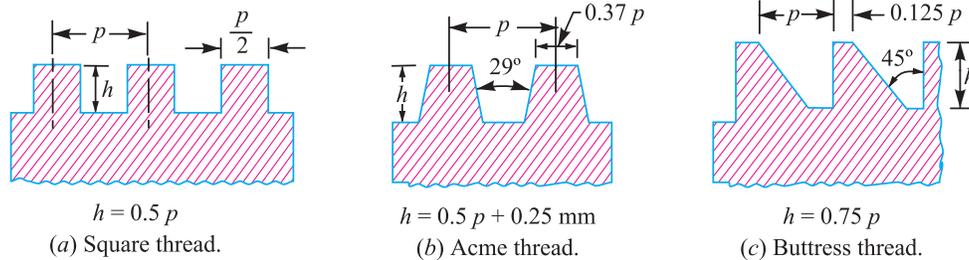


Fig. 17.1. Types of power screws.

pressure on the nut. It is difficult to cut with taps and dies. It is usually cut on a lathe with a single point tool and it can not be easily compensated for wear. The square threads are employed in screw jacks, presses and clamping devices. The standard dimensions for square threads according to IS : 4694 – 1968 (Reaffirmed 1996), are shown in Table 17.1 to 17.3.

2. Acme or trapezoidal thread. An acme or trapezoidal thread, as shown in Fig. 17.1 (b), is a modification of square thread. The slight slope given to its sides lowers the efficiency slightly than square thread and it also introduce some bursting pressure on the nut, but increases its area in shear. It is used where a split nut is required and where provision is made to take up wear as in the lead screw of a lathe. Wear may be taken up by means of an adjustable split nut. An acme thread may be cut by means of dies and hence it is more easily manufactured than square thread. The standard dimensions for acme or trapezoidal threads are shown in Table 17.4 (Page 630).

3. Buttress thread. A buttress thread, as shown in Fig. 17.1 (c), is used when large forces act along the screw axis in one direction only. This thread combines the higher efficiency of square thread and the ease of cutting and the adaptability to a split nut of acme thread. It is stronger than other threads because of greater thickness at the base of the thread. The buttress thread has limited use for power transmission. It is employed as the thread for light jack screws and vices.



Table 17.1. Basic dimensions for square threads in mm (Fine series) according to IS : 4694 – 1968 (Reaffirmed 1996)

Nominal diameter (d_1)	Major diameter		Minor diameter (d_c)	Pitch (p)	Depth of thread		Area of core (A_c) mm ²
	Bolt (d)	Nut (D)			Bolt (h)	Nut (H)	
10	10	10.5	8	2	1	1.25	50.3
12	12	12.5	10				78.5

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d_1	d	D	d_c	p	h	H	A_c
14	14	14.5	12	2	1	1.25	113
16	16	16.5	14				154
18	18	18.5	16				201
20	20	20.5	18				254
22	22	22.5	19	3	1.5	1.75	284
24	24	24.5	21				346
26	26	26.5	23				415
28	28	28.5	25				491
30	30	30.5	27				573
32	32	32.5	29				661
(34)	34	34.5	31				755
36	36	36.5	33				855
(38)	38	38.5	35				962
40	40	40.5	37				1075
42	42	42.5	39				1195
44	44	44.5	41				1320
(46)	46	46.5	43				1452
48	48	48.5	45				1590
50	50	50.5	47				1735
52	52	52.5	49				1886
55	55	55.5	52	2124			
(58)	58	58.5	55	2376			
60	60	60.5	57	2552			
(62)	62	62.5	59	2734			
65	65	65.5	61	4	2	2.25	2922
(68)	68	68.5	64				3217
70	70	70.5	66				3421
(72)	72	72.5	68				3632
75	75	75.5	71				3959
(78)	78	78.5	74				4301
80	80	80.5	76				4536
(82)	82	82.5	78				4778
(85)	85	85.5	81				5153
(88)	88	88.5	84				5542
90	90	90.5	86				5809
(92)	92	92.5	88				6082
95	95	95.5	91	6504			
(98)	98	98.5	94	6960			

d_1	d	D	d_c	p	h	H	A_c
100	100	100.5	96	4	2	2.25	7238
(105)	105	105.5	101				8012
110	110	110.5	106				8825
(115)	115	115.5	109	6	3	3.25	9331
120	120	120.5	114				10207
(125)	125	125.5	119				11 122
130	130	130.5	124				12 076
(135)	135	135.5	129				13 070
140	140	140.5	134				14 103
(145)	145	145.5	139				15 175
150	150	150.5	144				16 286
(155)	155	155.5	149				17437
160	160	160.5	154				18 627
(165)	165	165.5	159				19 856
170	170	170.5	164				21124
(175)	175	175.5	169				22 432

Note : Diameter within brackets are of second preference.

Table 17.2. Basic dimensions for square threads in mm (Normal series) according to IS : 4694 – 1968 (Reaffirmed 1996)

Nominal diameter (d_1)	Major diameter		Minor diameter (d_c)	Pitch (p)	Depth of thread		Area of core (A_c) mm ²
	Bolt (d)	Nut (D)			Bolt (h)	Nut (H)	
22	22	22.5	17	5	2.5	2.75	227
24	24	24.5	19				284
26	26	26.5	21				346
28	28	28.5	23				415
30	30	30.5	24	6	3	3.25	452
32	32	32.5	26				531
(34)	34	34.5	28				616
36	36	36.5	30				707
(38)	38	38.5	31	7	3.5	3.75	755
40	40	40.5	33				855
(42)	42	42.5	35				962
44	44	44.5	37				1075

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d_1	d	D	d_c	p	h	H	A_c
(46)	46	46.5	38	8	4	4.25	1134
48	48	48.5	40				1257
50	50	50.5	42				1385
52	52	52.5	44				1521
55	55	55.5	46	9	4.5	5.25	1662
(58)	58	58.5	49				1886
(60)	60	60.5	51				2043
(62)	62	62.5	53				2206
65	65	65.5	55	10	5	5.25	2376
(68)	68	68.5	58				2642
70	70	70.5	60				2827
(72)	72	72.5	62				3019
75	75	75.5	65				3318
(78)	78	78.5	68				3632
80	80	80.5	70				3848
(82)	82	82.5	72				4072
85	85	85.5	73	12	6	6.25	41.85
(88)	88	88.5	76				4536
90	90	85.5	78				4778
(92)	92	92.5	80				5027
95	95	95.5	83				5411
(98)	98	98.5	86				5809
100	100	100.5	88				6082
(105)	105	105.5	93				6793
110	110	110.5	98				7543
(115)	115	116	101	14	7	7.5	8012
120	120	121	106				882
(125)	125	126	111				9677
130	130	131	116				10 568
(135)	135	136	121				11 499
140	140	141	126				12 469
(145)	145	146	131				13 478
150	150	151	134				16
(155)	155	156	139	15 175			
160	160	161	144	16 286			

d_1	d	D	d_c	p	h	H	A_c
(165)	165	166	149				17 437
170	170	171	154	16	8	8.5	18 627
(175)	175	176	159				19 856

Note : Diameter within brackets are of second preference.

Table 17.3. Basic dimensions for square threads in mm (Coarse series) according to IS : 4694 – 1968 (Reaffirmed 1996)

Nominal diameter (d_1)	Major diameter		Minor diameter (d_c)	Pitch (p)	Depth of thread		Area of core (A_c) mm ²
	Bolt (d)	Nut (D)			Bolt (h)	Nut (H)	
22	22	22.5	14				164
24	24	24.5	16	8	4	4.25	204
26	26	26.5	18				254
28	28	28.5	20				314
30	30	30.5	20				314
32	32	32.5	22				380
(34)	34	34.5	24	10	5	5.25	452
36	36	36.5	26				531
(38)	38	38.5	28				616
40	40	40.5	28				616
(42)	42	42.5	30				707
44	44	44.5	32				804
(46)	46	46.5	34	12	6	6.25	908
48	48	48.5	36				1018
50	50	50.5	38				1134
52	52	52.5	40				1257
55	55	56	41				1320
(58)	58	59	44	14	7	7.25	1521
60	60	61	46				1662
(62)	62	63	48				1810
65	65	66	49				1886
(68)	68	69	52	16	8	8.5	2124
70	70	71	54				2290
(72)	72	73	56				2463
75	75	76	59				2734
(78)	78	79	62				3019
80	80	81	64				3217
(82)	82	83	66				3421

d_1	d	D	d_c	p	h	H	A_c
85	85	86	67				3526
(88)	88	89	70				3848
90	90	91	72				4072
(92)	92	93	74	18	9	9.5	4301
95	95	96	77				4657
(96)	96	99	80				5027
100	100	101	80				5027
(105)	105	106	85	20	10	10.5	5675
110	110	111	90				6362
(115)	115	116	93				6793
120	120	121	98				7543
(125)	125	126	103	22	11	11.5	8332
130	130	131	108				9161
(135)	135	136	111				9667
140	140	141	116	24	12	12.5	10 568
(145)	145	146	121				11 499
150	150	151	126				12 469
(155)	155	156	131				13 478
160	160	161	132				13 635
(165)	165	166	137				14 741
170	170	171	142	28	14	14.5	15 837
(175)	175	176	147				16 972

Note : Diameters within brackets are of second preference.

Table 17.4. Basic dimensions for trapezoidal/Acme threads.

Nominal or major diameter (d) mm.	Minor or core diameter (d_c) mm	Pitch (p) mm	Area of core (A_c) mm ²
10	6.5	3	33
12	8.5		57
14	9.5		71
16	11.5	4	105
18	13.5		143
20	15.5		189
22	16.5		214
24	18.5	5	269
26	20.5		330
28	22.5		389
30	23.5		434
32	25.5	6	511
34	27.5		594
36	29.5		683

d	d_c	p	A_c
38	30.5	7	731
40	32.5		830
42	34.5		935
44	36.5		1046
46	37.5	8	1104
48	39.5		1225
50	41.5		1353
52	43.5		1486
55	45.5	9	1626
58	48.5		1847
60	50.5		2003
62	52.5		2165
65	54.5	10	2333
68	57.5		2597
70	59.5		2781
72	61.5		2971
75	64.5		3267
78	67.5		3578
80	69.5		3794
82	71.5		4015
85	72.5	12	4128
88	75.5		4477
90	77.5		4717
92	79.5		4964
95	82.5		5346
98	85.5		5741
100	87.5		6013
105	92.5		6720
110	97.5		7466
115	100	7854	
120	105	14	8659
125	110		9503
130	115		10 387
135	120		11 310
140	125		12 272
145	130		13 273
150	133		13 893
155	138		16
160	143	16 061	
165	148	17 203	
170	153	18 385	
175	158	19 607	

17.3 Multiple Threads

The power screws with multiple threads such as double, triple etc. are employed when it is desired to secure a large lead with fine threads or high efficiency. Such type of threads are usually found in high speed actuators.

17.4 Torque Required to Raise Load by Square Threaded Screws

The torque required to raise a load by means of square threaded screw may be determined by considering a screw jack as shown in Fig. 17.2 (a). The load to be raised or lowered is placed on the head of the square threaded rod which is rotated by the application of an effort at the end of lever for lifting or lowering the load.

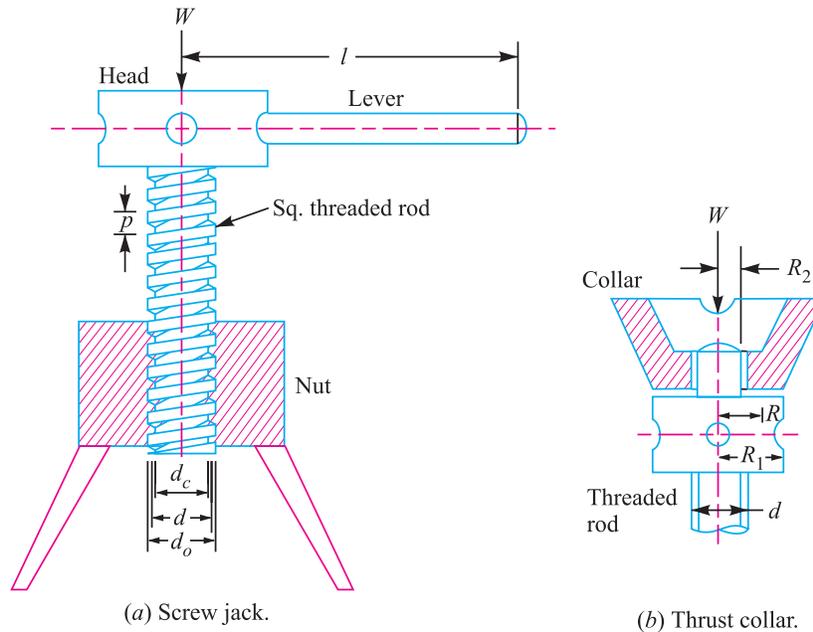


Fig. 17.2

A little consideration will show that if one complete turn of a screw thread be imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in Fig. 17.3 (a).

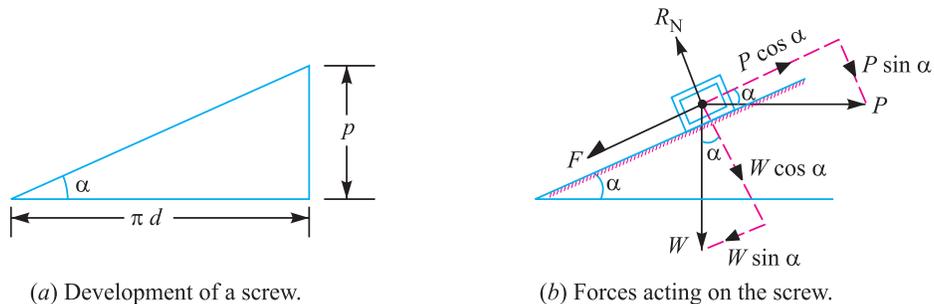


Fig. 17.3

- Let
- p = Pitch of the screw,
 - d = Mean diameter of the screw,
 - α = Helix angle,

P = Effort applied at the circumference of the screw to lift the load,
 W = Load to be lifted, and
 μ = Coefficient of friction, between the screw and nut
 = $\tan \phi$, where ϕ is the friction angle.

From the geometry of the Fig. 17.3 (a), we find that

$$\tan \alpha = p / \pi d$$

Since the principle, on which a screw jack works is similar to that of an inclined plane, therefore the force applied on the circumference of a screw jack may be considered to be horizontal as shown in Fig. 17.3 (b).

Since the load is being lifted, therefore the force of friction ($F = \mu.R_N$) will act downwards. All the forces acting on the body are shown in Fig. 17.3 (b).

Resolving the forces along the plane,

$$P \cos \alpha = W \sin \alpha + F = W \sin \alpha + \mu.R_N \quad \dots(i)$$

and resolving the forces perpendicular to the plane,

$$R_N = P \sin \alpha + W \cos \alpha \quad \dots(ii)$$

Substituting this value of R_N in equation (i), we have

$$\begin{aligned} P \cos \alpha &= W \sin \alpha + \mu (P \sin \alpha + W \cos \alpha) \\ &= W \sin \alpha + \mu P \sin \alpha + \mu W \cos \alpha \end{aligned}$$

or $P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$

or $P (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$

$$\therefore P = W \times \frac{(\sin \alpha + \mu \cos \alpha)}{(\cos \alpha - \mu \sin \alpha)}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we get

$$\text{or } P = W \times \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha}$$

Multiplying the numerator and denominator by $\cos \phi$, we have

$$\begin{aligned} P &= W \times \frac{\sin \alpha \cos \phi + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \alpha \sin \phi} \\ &= W \times \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)} = W \tan (\alpha + \phi) \end{aligned}$$

\therefore Torque required to overcome friction between the screw and nut,

$$T_1 = P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2}$$

When the axial load is taken up by a thrust collar as shown in Fig. 17.2 (b), so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,

$$\begin{aligned} T_2 &= \frac{2}{3} \times \mu_1 \times W \left[\frac{(R_1)^3 - (R_2)^3}{(R_1)^2 - (R_2)^2} \right] \quad \dots \text{(Assuming uniform pressure conditions)} \\ &= \mu_1 \times W \left(\frac{R_1 + R_2}{2} \right) = \mu_1 W R \quad \dots \text{(Assuming uniform wear conditions)} \end{aligned}$$

where

R_1 and R_2 = Outside and inside radii of collar,

R = Mean radius of collar = $\frac{R_1 + R_2}{2}$, and

μ_1 = Coefficient of friction for the collar.



Screw jack

∴ Total torque required to overcome friction (*i.e.* to rotate the screw),

$$T = T_1 + T_2$$

If an effort P_1 is applied at the end of a lever of arm length l , then the total torque required to overcome friction must be equal to the torque applied at the end of lever, *i.e.*

$$T = P \times \frac{d}{2} = P_1 \times l$$

Notes: 1. When the *nominal diameter (d_o) and the **core diameter (d_c) of the screw is given, then

Mean diameter of screw,
$$d = \frac{d_o + d_c}{2} = d_o - \frac{p}{2} = d_c + \frac{p}{2}$$

2. Since the mechanical advantage is the ratio of the load lifted (W) to the effort applied (P_1) at the end of the lever, therefore mechanical advantage,

$$\begin{aligned} \text{M.A.} &= \frac{W}{P_1} = \frac{W \times 2l}{P \times d} && \dots \left(\because P \times \frac{d}{2} = P_1 \times l \text{ or } P_1 = \frac{P \times d}{2l} \right) \\ &= \frac{W \times 2l}{W \tan(\alpha + \phi) d} = \frac{2l}{d \tan(\alpha + \phi)} \end{aligned}$$

17.5 Torque Required to Lower Load by Square Threaded Screws

A little consideration will show that when the load is being lowered, the force of friction ($F = \mu \cdot R_N$) will act upwards. All the forces acting on the body are shown in Fig. 17.4.

Resolving the forces along the plane,

$$\begin{aligned} P \cos \alpha &= F - W \sin \alpha \\ &= \mu R_N - W \sin \alpha \end{aligned} \quad \dots(i)$$

and resolving the forces perpendicular to the plane,

$$R_N = W \cos \alpha - P \sin \alpha \quad \dots(ii)$$

Substituting this value of R_N in equation (i), we have,

$$\begin{aligned} P \cos \alpha &= \mu (W \cos \alpha - P \sin \alpha) - W \sin \alpha \\ &= \mu W \cos \alpha - \mu P \sin \alpha - W \sin \alpha \end{aligned}$$

or
$$P \cos \alpha + \mu P \sin \alpha = \mu W \cos \alpha - W \sin \alpha$$

$$P (\cos \alpha + \mu \sin \alpha) = W (\mu \cos \alpha - \sin \alpha)$$

or
$$P = W \times \frac{(\mu \cos \alpha - \sin \alpha)}{(\cos \alpha + \mu \sin \alpha)}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we have

$$P = W \times \frac{(\tan \phi \cos \alpha - \sin \alpha)}{(\cos \alpha + \tan \phi \sin \alpha)}$$

Multiplying the numerator and denominator by $\cos \phi$, we have

$$\begin{aligned} P &= W \times \frac{(\sin \phi \cos \alpha - \cos \phi \sin \alpha)}{(\cos \phi \cos \alpha + \sin \phi \sin \alpha)} \\ &= W \times \frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)} = W \tan(\phi - \alpha) \end{aligned}$$

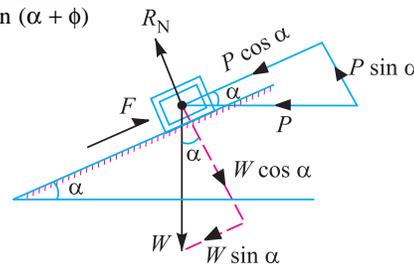


Fig. 17.4

* The nominal diameter of a screw thread is also known as *outside diameter* or *major diameter*.

** The core diameter of a screw thread is also known as *inner diameter* or *root diameter* or *minor diameter*.

∴ Torque required to overcome friction between the screw and nut,

$$T_1 = P \times \frac{d}{2} = W \tan (\phi - \alpha) \frac{d}{2}$$

Note : When $\alpha > \phi$, then $P = W \tan (\alpha - \phi)$.

17.6 Efficiency of Square Threaded Screws

The efficiency of square threaded screws may be defined as the ratio between the ideal effort (*i.e.* the effort required to move the load, neglecting friction) to the actual effort (*i.e.* the effort required to move the load taking friction into account).

We have seen in Art. 17.4 that the effort applied at the circumference of the screw to lift the load is

$$P = W \tan (\alpha + \phi) \quad \dots(i)$$

where

$W =$ Load to be lifted,

$\alpha =$ Helix angle,

$\phi =$ Angle of friction, and

$\mu =$ Coefficient of friction between the screw and nut = $\tan \phi$.

If there would have been no friction between the screw and the nut, then ϕ will be equal to zero. The value of effort P_0 necessary to raise the load, will then be given by the equation,

$$P_0 = W \tan \alpha \quad \text{[Substituting } \phi = 0 \text{ in equation (i)]}$$

$$\therefore \text{Efficiency, } \eta = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{P_0}{P} = \frac{W \tan \alpha}{W \tan (\alpha + \phi)} = \frac{\tan \alpha}{\tan (\alpha + \phi)}$$

This shows that the efficiency of a screw jack, is independent of the load raised.

In the above expression for efficiency, only the screw friction is considered. However, if the screw friction and collar friction is taken into account, then

$$\begin{aligned} \eta &= \frac{\text{Torque required to move the load, neglecting friction}}{\text{Torque required to move the load, including screw and collar friction}} \\ &= \frac{T_0}{T} = \frac{P_0 \times d / 2}{P \times d / 2 + \mu_1 . W . R} \end{aligned}$$

Note: The efficiency may also be defined as the ratio of mechanical advantage to the velocity ratio.

We know that mechanical advantage,

$$\text{M.A.} = \frac{W}{P_1} = \frac{W \times 2l}{P \times d} = \frac{W \times 2l}{W \tan (\alpha + \phi) d} = \frac{2l}{d \tan (\alpha + \phi)} \quad \dots(\text{Refer Art .17.4})$$

and velocity ratio,

$$\begin{aligned} \text{V.R.} &= \frac{\text{Distance moved by the effort } (P_1) \text{ in one revolution}}{\text{Distance moved by the load } (W) \text{ in one revolution}} \\ &= \frac{2 \pi l}{p} = \frac{2 \pi l}{\tan \alpha \times \pi d} = \frac{2 l}{d \tan \alpha} \quad \dots (\because \tan \alpha = p / \pi d) \end{aligned}$$

$$\therefore \text{Efficiency, } \eta = \frac{\text{M.A.}}{\text{V.R.}} = \frac{2 l}{d \tan (\alpha + \phi)} \times \frac{d \tan \alpha}{2 l} = \frac{\tan \alpha}{\tan (\alpha + \phi)}$$

17.7 Maximum Efficiency of a Square Threaded Screw

We have seen in Art. 17.6 that the efficiency of a square threaded screw,

$$\eta = \frac{\tan \alpha}{\tan (\alpha + \phi)} = \frac{\sin \alpha / \cos \alpha}{\sin (\alpha + \phi) / \cos (\alpha + \phi)} = \frac{\sin \alpha \times \cos (\alpha + \phi)}{\cos \alpha \times \sin (\alpha + \phi)} \quad \dots(i)$$

Multiplying the numerator and denominator by 2, we have,

$$\eta = \frac{2 \sin \alpha \times \cos (\alpha + \phi)}{2 \cos \alpha \times \sin (\alpha + \phi)} = \frac{\sin (2\alpha + \phi) - \sin \phi}{\sin (2\alpha + \phi) + \sin \phi} \quad \dots(ii)$$

$$\dots \left[\begin{array}{l} \because 2 \sin A \cos B = \sin (A + B) + \sin (A - B) \\ 2 \cos A \sin B = \sin (A + B) - \sin (A - B) \end{array} \right]$$

The efficiency given by equation (ii) will be maximum when $\sin (2\alpha + \phi)$ is maximum, i.e. when

$$\sin (2\alpha + \phi) = 1 \quad \text{or} \quad \text{when} \quad 2\alpha + \phi = 90^\circ$$

$$\therefore \quad 2\alpha = 90^\circ - \phi \quad \text{or} \quad \alpha = 45^\circ - \phi / 2$$

Substituting the value of 2α in equation (ii), we have maximum efficiency,

$$\eta_{max} = \frac{\sin (90^\circ - \phi + \phi) - \sin \phi}{\sin (90^\circ - \phi + \phi) + \sin \phi} = \frac{\sin 90^\circ - \sin \phi}{\sin 90^\circ + \sin \phi} = \frac{1 - \sin \phi}{1 + \sin \phi}$$

Example 17.1. A vertical screw with single start square threads of 50 mm mean diameter and 12.5 mm pitch is raised against a load of 10 kN by means of a hand wheel, the boss of which is threaded to act as a nut. The axial load is taken up by a thrust collar which supports the wheel boss and has a mean diameter of 60 mm. The coefficient of friction is 0.15 for the screw and 0.18 for the collar. If the tangential force applied by each hand to the wheel is 100 N, find suitable diameter of the hand wheel.

Solution. Given : $d = 50 \text{ mm}$; $p = 12.5 \text{ mm}$; $W = 10 \text{ kN} = 10 \times 10^3 \text{ N}$; $D = 60 \text{ mm}$ or $R = 30 \text{ mm}$; $\mu = \tan \phi = 0.15$; $\mu_1 = 0.18$; $P_1 = 100 \text{ N}$

$$\text{We know that } \tan \alpha = \frac{p}{\pi d} = \frac{12.5}{\pi \times 50} = 0.08$$

and the tangential force required at the circumference of the screw,

$$P = W \tan (\alpha + \phi) = W \left(\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right)$$

$$= 10 \times 10^3 \left[\frac{0.08 + 0.15}{1 - 0.08 \times 0.15} \right] = 2328 \text{ N}$$

We also know that the total torque required to turn the hand wheel,

$$T = P \times \frac{d}{2} + \mu_1 W R = 2328 \times \frac{50}{2} + 0.18 \times 10 \times 10^3 \times 30 \text{ N-mm}$$

$$= 58\,200 + 54\,000 = 112\,200 \text{ N-mm} \quad \dots(i)$$

Let $D_1 =$ Diameter of the hand wheel in mm.

We know that the torque applied to the handwheel,

$$T = 2 P_1 \times \frac{D_1}{2} = 2 \times 100 \times \frac{D_1}{2} = 100 D_1 \text{ N-mm} \quad \dots(ii)$$

Equating equations (i) and (ii),

$$D_1 = 112\,200 / 100 = 1122 \text{ mm} = 1.122 \text{ m} \text{ Ans.}$$

Example 17.2. An electric motor driven power screw moves a nut in a horizontal plane against a force of 75 kN at a speed of 300 mm / min. The screw has a single square thread of 6 mm pitch on a major diameter of 40 mm. The coefficient of friction at screw threads is 0.1. Estimate power of the motor.

Solution. Given : $W = 75 \text{ kN} = 75 \times 10^3 \text{ N}$; $v = 300 \text{ mm/min}$; $p = 6 \text{ mm}$; $d_o = 40 \text{ mm}$; $\mu = \tan \phi = 0.1$

We know that mean diameter of the screw,

$$d = d_o - p / 2 = 40 - 6 / 2 = 37 \text{ mm}$$

and

$$\tan \alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 37} = 0.0516$$

We know that tangential force required at the circumference of the screw,

$$\begin{aligned} P &= W \tan (\alpha + \phi) = W \left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right] \\ &= 75 \times 10^3 \left[\frac{0.0516 + 0.1}{1 - 0.0516 \times 0.1} \right] = 11.43 \times 10^3 \text{ N} \end{aligned}$$

and torque required to operate the screw,

$$T = P \times \frac{d}{2} = 11.43 \times 10^3 \times \frac{37}{2} = 211.45 \times 10^3 \text{ N-mm} = 211.45 \text{ N-m}$$

Since the screw moves in a nut at a speed of 300 mm / min and the pitch of the screw is 6 mm, therefore speed of the screw in revolutions per minute (r.p.m.),

$$N = \frac{\text{Speed in mm/min.}}{\text{Pitch in mm}} = \frac{300}{6} = 50 \text{ r.p.m.}$$

and angular speed,

$$\omega = 2\pi N / 60 = 2\pi \times 50 / 60 = 5.24 \text{ rad /s}$$

$$\therefore \text{Power of the motor} = T \cdot \omega = 211.45 \times 5.24 = 1108 \text{ W} = 1.108 \text{ kW} \text{ Ans.}$$

Example. 17.3. The cutter of a broaching machine is pulled by square threaded screw of 55 mm external diameter and 10 mm pitch. The operating nut takes the axial load of 400 N on a flat surface of 60 mm and 90 mm internal and external diameters respectively. If the coefficient of friction is 0.15 for all contact surfaces on the nut, determine the power required to rotate the operating nut when the cutting speed is 6 m/min. Also find the efficiency of the screw.

Solution. Given : $d_o = 55 \text{ mm}$; $p = 10 \text{ mm} = 0.01 \text{ m}$; $W = 400 \text{ N}$; $D_2 = 60 \text{ mm}$ or $R_2 = 30 \text{ mm}$; $D_1 = 90 \text{ mm}$ or $R_1 = 45 \text{ mm}$; $\mu = \tan \phi = \mu_1 = 0.15$; Cutting speed = 6 m / min

Power required to operate the nut

We know that the mean diameter of the screw,

$$d = d_o - p / 2 = 55 - 10 / 2 = 50 \text{ mm}$$

$$\therefore \tan \alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0637$$

and force required at the circumference of the screw,

$$\begin{aligned} P &= W \tan (\alpha + \phi) = W \left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right] \\ &= 400 \left[\frac{0.0637 + 0.15}{1 - 0.0637 \times 0.15} \right] = 86.4 \text{ N} \end{aligned}$$

We know that mean radius of the flat surface,

$$R = \frac{R_1 + R_2}{2} = \frac{45 + 30}{2} = 37.5 \text{ mm}$$

\therefore Total torque required,

$$\begin{aligned} T &= P \times \frac{d}{2} + \mu_1 W R = 86.4 \times \frac{50}{2} + 0.15 \times 400 \times 37.5 \text{ N-mm} \\ &= 4410 \text{ N-mm} = 4.41 \text{ N-m} \end{aligned}$$

We know that speed of the screw,

$$N = \frac{\text{Cutting speed}}{\text{Pitch}} = \frac{6}{0.01} = 600 \text{ r.p.m}$$

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and angular speed, $\omega = 2\pi N / 60 = 2\pi \times 600 / 60 = 62.84 \text{ rad / s}$

\therefore Power required to operate the nut

$$= T \cdot \omega = 4.41 \times 62.84 = 277 \text{ W} = 0.277 \text{ kW} \quad \text{Ans.}$$

Efficiency of the screw

We know that the efficiency of the screw,

$$\begin{aligned} \eta &= \frac{T_0}{T} = \frac{W \tan \alpha \times d / 2}{T} = \frac{400 \times 0.0637 \times 50 / 2}{4410} \\ &= 0.144 \quad \text{or} \quad 14.4\% \quad \text{Ans.} \end{aligned}$$

Example 17.4. A vertical two start square threaded screw of a 100 mm mean diameter and 20 mm pitch supports a vertical load of 18 kN. The axial thrust on the screw is taken by a collar bearing of 250 mm outside diameter and 100 mm inside diameter. Find the force required at the end of a lever which is 400 mm long in order to lift and lower the load. The coefficient of friction for the vertical screw and nut is 0.15 and that for collar bearing is 0.20.

Solution. Given : $d = 100 \text{ mm}$; $p = 20 \text{ mm}$; $W = 18 \text{ kN} = 18 \times 10^3 \text{ N}$; $D_1 = 250 \text{ mm}$ or $R_1 = 125 \text{ mm}$; $D_2 = 100 \text{ mm}$ or $R_2 = 50 \text{ mm}$; $l = 400 \text{ mm}$; $\mu = \tan \phi = 0.15$; $\mu_1 = 0.20$

Force required at the end of lever

Let $P =$ Force required at the end of lever.

Since the screw is a two start square threaded screw, therefore lead of the screw

$$= 2p = 2 \times 20 = 40 \text{ mm}$$

We know that $\tan \alpha = \frac{\text{Lead}}{\pi d} = \frac{40}{\pi \times 100} = 0.127$

1. For raising the load

We know that tangential force required at the circumference of the screw,

$$\begin{aligned} P &= W \tan (\alpha + \phi) = W \left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right] \\ &= 18 \times 10^3 \left[\frac{0.127 + 0.15}{1 - 0.127 \times 0.15} \right] = 5083 \text{ N} \end{aligned}$$

and mean radius of the collar,

$$R = \frac{R_1 + R_2}{2} = \frac{125 + 50}{2} = 87.5 \text{ mm}$$

\therefore Total torque required at the end of lever,

$$\begin{aligned} T &= P \times \frac{d}{2} + \mu_1 WR \\ &= 5083 \times \frac{100}{2} + 0.20 \times 18 \times 10^3 \times 87.5 = 569 \text{ 150 N-mm} \end{aligned}$$

We know that torque required at the end of lever (T),

$$569 \text{ 150} = P_1 \times l = P_1 \times 400 \quad \text{or} \quad P_1 = 569 \text{ 150} / 400 = 1423 \text{ N} \quad \text{Ans.}$$

2. For lowering the load

We know that tangential force required at the circumference of the screw,

$$\begin{aligned} P &= W \tan (\phi - \alpha) = W \left[\frac{\tan \phi - \tan \alpha}{1 + \tan \phi \tan \alpha} \right] \\ &= 18 \times 10^3 \left[\frac{0.15 - 0.127}{1 + 0.15 \times 0.127} \right] = 406.3 \text{ N} \end{aligned}$$

and the total torque required the end of lever,

$$T = P \times \frac{d}{2} + \mu_1 W R$$

$$= 406.3 \times \frac{100}{2} + 0.20 \times 18 \times 10^3 \times 87.5 = 335\,315 \text{ N-mm}$$

We know that torque required at the end of lever (T),

$$335\,315 = P_1 \times l = P_1 \times 400 \quad \text{or} \quad P_1 = 335\,315 / 400 = 838.3 \text{ N Ans.}$$

Example 17.5. The mean diameter of the square threaded screw having pitch of 10 mm is 50 mm. A load of 20 kN is lifted through a distance of 170 mm. Find the work done in lifting the load and the efficiency of the screw, when

1. The load rotates with the screw, and
2. The load rests on the loose head which does not rotate with the screw.

The external and internal diameter of the bearing surface of the loose head are 60 mm and 10 mm respectively. The coefficient of friction for the screw and the bearing surface may be taken as 0.08.

Solution. Given : $p = 10 \text{ mm}$; $d = 50 \text{ mm}$; $W = 20 \text{ kN} = 20 \times 10^3 \text{ N}$; $D_1 = 60 \text{ mm}$ or $R_2 = 30 \text{ mm}$; $D_2 = 10 \text{ mm}$ or $R_2 = 5 \text{ mm}$; $\mu = \tan \phi = \mu_1 = 0.08$

We know that $\tan \alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0637$

∴ Force required at the circumference of the screw to lift the load,

$$P = W \tan (\alpha + \phi) = W \left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right]$$

$$= 20 \times 10^3 \left[\frac{0.0637 + 0.08}{1 - 0.0637 \times 0.08} \right] = 2890 \text{ N}$$

and torque required to overcome friction at the screw,

$$T = P \times d / 2 = 2890 \times 50 / 2 = 72\,250 \text{ N-mm} = 72.25 \text{ N-m}$$

Since the load is lifted through a vertical distance of 170 mm and the distance moved by the screw in one rotation is 10 mm (equal to pitch), therefore number of rotations made by the screw,

$$N = 170 / 10 = 17$$

1. When the load rotates with the screw

We know that workdone in lifting the load

$$= T \times 2 \pi N = 72.25 \times 2\pi \times 17 = 7718 \text{ N-m Ans.}$$

and efficiency of the screw,

$$\eta = \frac{\tan \alpha}{\tan (\alpha + \phi)} = \frac{\tan \alpha (1 - \tan \alpha \tan \phi)}{\tan \alpha + \tan \phi}$$

$$= \frac{0.0637 (1 - 0.0637 \times 0.08)}{0.0637 + 0.08} = 0.441 \text{ or } 44.1\% \quad \text{Ans.}$$

2. When the load does not rotate with the screw

We know that mean radius of the bearing surface,

$$R = \frac{R_1 + R_2}{2} = \frac{30 + 5}{2} = 17.5 \text{ mm}$$

and torque required to overcome friction at the screw and the collar,

$$T = P \times \frac{d}{2} + \mu_1 W R$$

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$$\begin{aligned}
 &= 2890 \times \frac{50}{2} + 0.08 \times 20 \times 10^3 \times 17.5 = 100\,250 \text{ N-mm} \\
 &= 100.25 \text{ N-m}
 \end{aligned}$$

∴ Workdone by the torque in lifting the load

$$= T \times 2\pi N = 100.25 \times 2\pi \times 17 = 10\,710 \text{ N-m Ans.}$$

We know that torque required to lift the load, neglecting friction,

$$\begin{aligned}
 T_0 &= P_0 \times d/2 = W \tan \alpha \times d/2 && \dots (P_0 = W \tan \alpha) \\
 &= 20 \times 10^3 \times 0.0637 \times 50/2 = 31\,850 \text{ N-mm} = 31.85 \text{ N-m}
 \end{aligned}$$

∴ Efficiency of the screw,

$$\eta = \frac{T_0}{T} = \frac{31.85}{100.25} = 0.318 \text{ or } 31.8\% \text{ Ans.}$$

17.8 Efficiency Vs Helix Angle

We have seen in Art. 17.6 that the efficiency of a square threaded screw depends upon the helix angle α and the friction angle ϕ . The variation of efficiency of a square threaded screw for raising the load with the helix angle α is shown in Fig. 17.5. We see that the efficiency of a square threaded screw increases rapidly upto helix angle of 20° , after which the increase in efficiency is slow. The efficiency is maximum for helix angle between 40 to 45° .

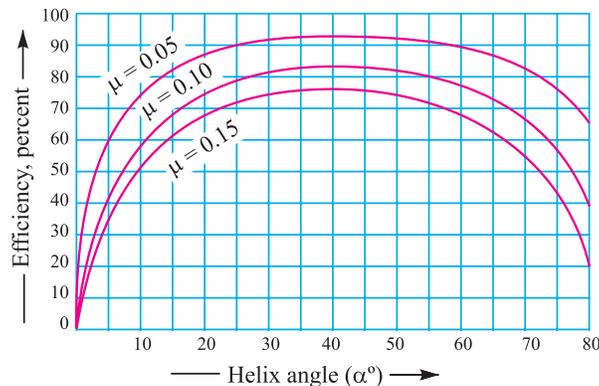


Fig. 17.5. Graph between efficiency and helix angle.

When the helix angle further increases say 70° , the efficiency drops. This is due to the fact that the normal thread force becomes large and thus the force of friction and the work of friction becomes large as compared with the useful work. This results in low efficiency.

17.9 Over Hauling and Self Locking Screws

We have seen in Art. 17.5 that the effort required at the circumference of the screw to lower the load is

$$P = W \tan (\phi - \alpha)$$

and the torque required to lower the load,

$$T = P \times \frac{d}{2} = W \tan (\phi - \alpha) \frac{d}{2}$$

In the above expression, if $\phi < \alpha$, then torque required to lower the load will be **negative**. In other words, the load will start moving downward without the application of any torque. Such a condition is known as **over hauling of screws**. If however, $\phi > \alpha$, the torque required to lower the load will be **positive**, indicating that an effort is applied to lower the load. Such a screw is known as



Mechanical power screw driver

self locking screw. In other words, a screw will be self locking if the friction angle is greater than helix angle or coefficient of friction is greater than tangent of helix angle *i.e.* μ or $\tan \phi > \tan \alpha$.

17.10 Efficiency of Self Locking Screws

We know that the efficiency of screw,

$$\eta = \frac{\tan \phi}{\tan (\alpha + \phi)}$$

and for self locking screws, $\phi \geq \alpha$ or $\alpha \leq \phi$.

∴ Efficiency for self locking screws,

$$\eta \leq \frac{\tan \phi}{\tan (\phi + \phi)} \leq \frac{\tan \phi}{\tan 2\phi} \leq \frac{\tan \phi (1 - \tan^2 \phi)}{2 \tan \phi} \leq \frac{1}{2} - \frac{\tan^2 \phi}{2}$$

$$\dots \left(\because \tan 2\phi = \frac{2 \tan \phi}{1 - \tan^2 \phi} \right)$$

From this expression we see that efficiency of self locking screws is less than $\frac{1}{2}$ or 50%. If the efficiency is more than 50%, then the screw is said to be overhauling.

Note: It can be proved as follows:

Let W = Load to be lifted, and
 h = Distance through which the load is lifted.

∴ Output = $W.h$

and Input = $\frac{\text{Output}}{\eta} = \frac{W.h}{\eta}$

∴ Work lost in overcoming friction

$$= \text{Input} - \text{Output} = \frac{W.h}{\eta} - W.h = W.h \left(\frac{1}{\eta} - 1 \right)$$

For self locking,

$$W.h \left(\frac{1}{\eta} - 1 \right) \leq W.h$$

∴ $\frac{1}{\eta} - 1 \leq 1$ or $\eta \leq \frac{1}{2}$ or 50%

17.11 Coefficient of Friction

The coefficient of friction depends upon various factors like *material of screw and nut, workmanship in cutting screw, quality of lubrication, unit bearing pressure and the rubbing speeds. The value of coefficient of friction does not vary much with different combination of material, load or rubbing speed, except under starting conditions. The coefficient of friction, with good lubrication and average workmanship, may be assumed between 0.10 and 0.15. The various values for coefficient of friction for steel screw and cast iron or bronze nut, under different conditions are shown in the following table.

Table 17.5. Coefficient of friction under different conditions.

S.No.	Condition	Average coefficient of friction	
		Starting	Running
1.	High grade materials and workmanship and best running conditions.	0.14	0.10
2.	Average quality of materials and workmanship and average running conditions.	0.18	0.13
3.	Poor workmanship or very slow and in frequent motion with indifferent lubrication or newly machined surface.	0.21	0.15

If the thrust collars are used, the values of coefficient of friction may be taken as shown in the following table.

Table 17.6. Coefficient of friction when thrust collars are used.

S.No.	Materials	Average coefficient of friction	
		Starting	Running
1.	Soft steel on cast iron	0.17	0.12
2.	Hardened steel on cast iron	0.15	0.09
3.	Soft steel on bronze	0.10	0.08
4.	Hardened steel on bronze	0.08	0.06

17.12 Acme or Trapezoidal Threads

We know that the normal reaction in case of a square threaded screw is

$$R_N = W \cos \alpha,$$

where α is the helix angle.

But in case of Acme or trapezoidal thread, the normal reaction between the screw and nut is increased because the axial component of this normal reaction must be equal to the axial load (W).

Consider an Acme or trapezoidal thread as shown in Fig. 17.6.

Let $**2\beta = \text{Angle of the Acme thread, and}$
 $\beta = \text{Semi-angle of the thread.}$

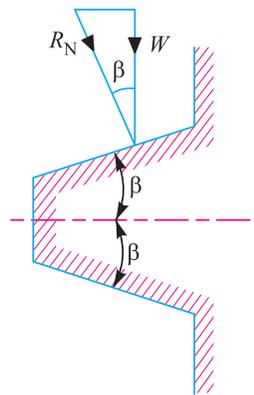


Fig. 17.6. Acme or trapezoidal threads.

* The material of screw is usually steel and the nut is made of cast iron, gun metal, phosphor bronze in order to keep the wear to a minimum.

** For Acme threads, $2\beta = 29^\circ$, and for trapezoidal threads, $2\beta = 30^\circ$.

$$\therefore R_N = \frac{W}{\cos \beta}$$

and frictional force, $F = \mu \cdot R_N = \mu \times \frac{W}{\cos \beta} = \mu_1 \cdot W$

where $\mu / \cos \beta = \mu_1$, known as **virtual coefficient of friction**.

Notes : 1. When coefficient of friction, $\mu_1 = \frac{\mu}{\cos \beta}$ is considered, then the Acme thread is equivalent to a square thread.

2. All equations of square threaded screw also hold good for Acme threads. In case of Acme threads, μ_1 (i.e. $\tan \phi_1$) may be substituted in place of μ (i.e. $\tan \phi$). Thus for Acme threads,

$$P = W \tan (\alpha + \phi_1)$$

where $\phi_1 =$ Virtual friction angle, and $\tan \phi_1 = \mu_1$.

Example 17.6. The lead screw of a lathe has Acme threads of 50 mm outside diameter and 8 mm pitch. The screw must exert an axial pressure of 2500 N in order to drive the tool carriage. The thrust is carried on a collar 110 mm outside diameter and 55 mm inside diameter and the lead screw rotates at 30 r.p.m. Determine (a) the power required to drive the screw; and (b) the efficiency of the lead screw. Assume a coefficient of friction of 0.15 for the screw and 0.12 for the collar.

Solution. Given : $d_o = 50$ mm ; $p = 8$ mm ; $W = 2500$ N ; $D_1 = 110$ mm or $R_1 = 55$ mm ; $D_2 = 55$ mm or $R_2 = 27.5$ mm ; $N = 30$ r.p.m. ; $\mu = \tan \phi = 0.15$; $\mu_2 = 0.12$

(a) Power required to drive the screw

We know that mean diameter of the screw,

$$d = d_o - p / 2 = 50 - 8 / 2 = 46 \text{ mm}$$

$$\therefore \tan \alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 46} = 0.055$$

Since the angle for Acme threads is $2\beta = 29^\circ$ or $\beta = 14.5^\circ$, therefore virtual coefficient of friction,

$$\mu_1 = \tan \phi_1 = \frac{\mu}{\cos \beta} = \frac{0.15}{\cos 14.5^\circ} = \frac{0.15}{0.9681} = 0.155$$

We know that the force required to overcome friction at the screw,

$$\begin{aligned} P &= W \tan (\alpha + \phi_1) = W \left[\frac{\tan \alpha + \tan \phi_1}{1 - \tan \alpha \tan \phi_1} \right] \\ &= 2500 \left[\frac{0.055 + 0.155}{1 - 0.055 \times 0.155} \right] = 530 \text{ N} \end{aligned}$$

and torque required to overcome friction at the screw.

$$T_1 = P \times d / 2 = 530 \times 46 / 2 = 12\,190 \text{ N-mm}$$

We know that mean radius of collar,

$$R = \frac{R_1 + R_2}{2} = \frac{55 + 27.5}{2} = 41.25 \text{ mm}$$

Assuming uniform wear, the torque required to overcome friction at collars,

$$T_2 = \mu_2 W R = 0.12 \times 2500 \times 41.25 = 12\,375 \text{ N-mm}$$

\therefore Total torque required to overcome friction,

$$T = T_1 + T_2 = 12\,190 + 12\,375 = 24\,565 \text{ N-mm} = 24.565 \text{ N-m}$$

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We know that power required to drive the screw

$$= T \cdot \omega = \frac{T \times 2 \pi N}{60} = \frac{24.565 \times 2 \pi \times 30}{60} = 77 \text{ W} = 0.077 \text{ kW} \quad \text{Ans.}$$

... ($\because \omega = 2\pi N / 60$)

(b) Efficiency of the lead screw

We know that the torque required to drive the screw with no friction,

$$T_o = W \tan \alpha \times \frac{d}{2} = 2500 \times 0.055 \times \frac{46}{2} = 3163 \text{ N-mm} = 3.163 \text{ N-m}$$

\therefore Efficiency of the lead screw,

$$\eta = \frac{T_o}{T} = \frac{3.163}{24.565} = 0.13 \text{ or } 13\% \quad \text{Ans.}$$

17.13 Stresses in Power Screws

A power screw must have adequate strength to withstand axial load and the applied torque. Following types of stresses are induced in the screw.

1. Direct tensile or compressive stress due to an axial load. The direct stress due to the axial load may be determined by dividing the axial load (W) by the minimum cross-sectional area of the screw (A_c) i.e. area corresponding to minor or core diameter (d_c).

\therefore Direct stress (tensile or compressive)

$$= \frac{W}{A_c}$$

This is only applicable when the axial load is compressive and the unsupported length of the screw between the load and the nut is short. But when the screw is axially loaded in compression and the unsupported length of the screw between the load and the nut is too great, then the design must be based on column theory assuming suitable end conditions. In such cases, the cross-sectional area corresponding to core diameter may be obtained by using Rankine-Gordon formula or J.B. Johnson's formula. According to this,

$$W_{cr} = A_c \times \sigma_y \left[1 - \frac{\sigma_y}{4 C \pi^2 E} \left(\frac{L}{k} \right)^2 \right]$$

$$\therefore \sigma_c = \frac{W}{A_c} \left[\frac{1}{1 - \frac{\sigma_y}{4 C \pi^2 E} \left(\frac{L}{k} \right)^2} \right]$$

where

W_{cr} = Critical load,

σ_y = Yield stress,

L = Length of screw,

k = Least radius of gyration,

C = End-fixity coefficient,

E = Modulus of elasticity, and

σ_c = Stress induced due to load W .

Note : In actual practice, the core diameter is first obtained by considering the screw under simple compression and then checked for critical load or buckling load for stability of the screw.

2. Torsional shear stress. Since the screw is subjected to a twisting moment, therefore torsional shear stress is induced. This is obtained by considering the minimum cross-section of the screw. We know that torque transmitted by the screw,

$$T = \frac{\pi}{16} \times \tau (d_c)^3$$

or shear stress induced,

$$\tau = \frac{16 T}{\pi (d_c)^3}$$

When the screw is subjected to both direct stress and torsional shear stress, then the design must be based on maximum shear stress theory, according to which maximum shear stress on the minor diameter section,

$$\tau_{max} = \frac{1}{2} \sqrt{(\sigma_t \text{ or } \sigma_c)^2 + 4 \tau^2}$$

It may be noted that when the unsupported length of the screw is short, then failure will take place when the maximum shear stress is equal to the shear yield strength of the material. In this case, shear yield strength,

$$\tau_y = \tau_{max} \times \text{Factor of safety}$$

3. Shear stress due to axial load. The threads of the screw at the core or root diameter and the threads of the nut at the major diameter may shear due to the axial load. Assuming that the load is uniformly distributed over the threads in contact, we have

Shear stress for screw,

$$\tau_{(screw)} = \frac{W}{\pi n \cdot d_c \cdot t}$$

and shear stress for nut,

$$\tau_{(nut)} = \frac{W}{\pi n \cdot d_o \cdot t}$$

where W = Axial load on the screw,

n = Number of threads in engagement,

d_c = Core or root diameter of the screw,

d_o = Outside or major diameter of nut or screw, and

t = Thickness or width of thread.



Friction between the threads of screw and nut plays important role in determining the efficiency and locking properties of a screw

4. Bearing pressure. In order to reduce wear of the screw and nut, the bearing pressure on the thread surfaces must be within limits. In the design of power screws, the bearing pressure depends upon the materials of the screw and nut, relative velocity between the nut and screw and the nature of lubrication. Assuming that the load is uniformly distributed over the threads in contact, the bearing pressure on the threads is given by

$$p_b = \frac{W}{\frac{\pi}{4} [(d_o)^2 - (d_c)^2] n} = \frac{*W}{\pi d \cdot t \cdot n}$$

where d = Mean diameter of screw,

t = Thickness or width of screw = $p / 2$, and

n = Number of threads in contact with the nut

$$= \frac{\text{Height of the nut}}{\text{Pitch of threads}} = \frac{h}{p}$$

Therefore, from the above expression, the height of nut or the length of thread engagement of the screw and nut may be obtained.

The following table shows some limiting values of bearing pressures.

* We know that $\frac{(d_o)^2 - (d_c)^2}{4} = \frac{d_o + d_c}{2} \times \frac{d_o - d_c}{2} = d \times \frac{p}{2} = d t$

Table 17.7. Limiting values of bearing pressures.

Application of screw	Material		Safe bearing pressure in N/mm^2	Rubbing speed at thread pitch diameter
	Screw	Nut		
1. Hand press	Steel	Bronze	17.5 - 24.5	Low speed, well lubricated
2. Screw jack	Steel	Cast iron	12.6 - 17.5	Low speed < 2.4 m / min
	Steel	Bronze	11.2 - 17.5	Low speed < 3 m / min
3. Hoisting screw	Steel	Cast iron	4.2 - 7.0	Medium speed 6 - 12 m / min
	Steel	Bronze	5.6 - 9.8	Medium speed 6 - 12 m / min
4. Lead screw	Steel	Bronze	1.05 - 1.7	High speed > 15 m / min

Example 17.7. A power screw having double start square threads of 25 mm nominal diameter and 5 mm pitch is acted upon by an axial load of 10 kN. The outer and inner diameters of screw collar are 50 mm and 20 mm respectively. The coefficient of thread friction and collar friction may be assumed as 0.2 and 0.15 respectively. The screw rotates at 12 r.p.m. Assuming uniform wear condition at the collar and allowable thread bearing pressure of $5.8 N/mm^2$, find: 1. the torque required to rotate the screw; 2. the stress in the screw; and 3. the number of threads of nut in engagement with screw.

Solution. Given : $d_o = 25$ mm ; $p = 5$ mm ; $W = 10$ kN = 10×10^3 N ; $D_1 = 50$ mm or $R_1 = 25$ mm ; $D_2 = 20$ mm or $R_2 = 10$ mm ; $\mu = \tan \phi = 0.2$; $\mu_1 = 0.15$; $N = 12$ r.p.m. ; $p_b = 5.8 N/mm^2$

1. Torque required to rotate the screw

We know that mean diameter of the screw,

$$d = d_o - p / 2 = 25 - 5 / 2 = 22.5 \text{ mm}$$

Since the screw is a double start square threaded screw, therefore lead of the screw,

$$= 2p = 2 \times 5 = 10 \text{ mm}$$

$$\therefore \tan \alpha = \frac{\text{Lead}}{\pi d} = \frac{10}{\pi \times 22.5} = 0.1414$$

We know that tangential force required at the circumference of the screw,

$$\begin{aligned} P &= W \tan (\alpha + \phi) = W \left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right] \\ &= 10 \times 10^3 \left[\frac{0.1414 + 0.2}{1 - 0.1414 \times 0.2} \right] = 3513 \text{ N} \end{aligned}$$

and mean radius of the screw collar,

$$R = \frac{R_1 + R_2}{2} = \frac{25 + 10}{2} = 17.5$$

∴ Total torque required to rotate the screw,

$$T = P \times \frac{d}{2} + \mu_1 W R = 3513 \times \frac{22.5}{2} + 0.15 \times 10 \times 10^3 \times 17.5 \text{ N-mm}$$

$$= 65\,771 \text{ N-mm} = 65.771 \text{ N-m Ans.}$$

2. Stress in the screw

We know that the inner diameter or core diameter of the screw,

$$d_c = d_o - p = 25 - 5 = 20 \text{ mm}$$

∴ Corresponding cross-sectional area of the screw,

$$A_c = \frac{\pi}{4} (d_c)^2 = \frac{\pi}{4} (20)^2 = 314.2 \text{ mm}^2$$

We know that direct stress,

$$\sigma_c = \frac{W}{A_c} = \frac{10 \times 10^3}{314.2} = 31.83 \text{ N/mm}^2$$

and shear stress,
$$\tau = \frac{16 T}{\pi (d_c)^3} = \frac{16 \times 65\,771}{\pi (20)^3} = 41.86 \text{ N/mm}^2$$

We know that maximum shear stress in the screw,

$$\tau_{max} = \frac{1}{2} \sqrt{(\sigma_c)^2 + 4\tau^2} = \frac{1}{2} \sqrt{(31.83)^2 + 4(41.86)^2}$$

$$= 44.8 \text{ N/mm}^2 = 44.8 \text{ MPa Ans.}$$

3. Number of threads of nut in engagement with screw

Let n = Number of threads of nut in engagement with screw, and

$$t = \text{Thickness of threads} = p / 2 = 5 / 2 = 2.5 \text{ mm}$$

We know that bearing pressure on the threads (p_b),

$$5.8 = \frac{W}{\pi d \times t \times n} = \frac{10 \times 10^3}{\pi \times 22.5 \times 2.5 \times n} = \frac{56.6}{n}$$

$$\therefore n = 56.6 / 5.8 = 9.76 \text{ say } 10 \text{ Ans.}$$

Example 17.8. The screw of a shaft straightener exerts a load of 30 kN as shown in Fig. 17.7. The screw is square threaded of outside diameter 75 mm and 6 mm pitch. Determine:

1. Force required at the rim of a 300 mm diameter hand wheel, assuming the coefficient of friction for the threads as 0.12;
2. Maximum compressive stress in the screw, bearing pressure on the threads and maximum shear stress in threads; and
3. Efficiency of the straightner.

Solution. Given : $W = 30 \text{ kN} = 30 \times 10^3 \text{ N}$; $d_o = 75 \text{ mm}$; $p = 6 \text{ mm}$; $D = 300 \text{ mm}$; $\mu = \tan \phi = 0.12$

1. Force required at the rim of handwheel

Let P_1 = Force required at the rim of handwheel.

We know that the inner diameter or core diameter of the screw,

$$d_c = d_o - p = 75 - 6 = 69 \text{ mm}$$

Mean diameter of the screw,

$$*d = \frac{d_o + d_c}{2} = \frac{75 + 69}{2} = 72 \text{ mm}$$

and

$$\tan \alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 72} = 0.0265$$

∴ Torque required to overcome friction at the threads,

$$\begin{aligned} T &= P \times \frac{d}{2} \\ &= W \tan(\alpha + \phi) \frac{d}{2} \\ &= W \left(\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right) \frac{d}{2} \\ &= 30 \times 10^3 \left(\frac{0.0265 + 0.12}{1 - 0.0265 \times 0.12} \right) \frac{72}{2} \\ &= 158\,728 \text{ N-mm} \end{aligned}$$

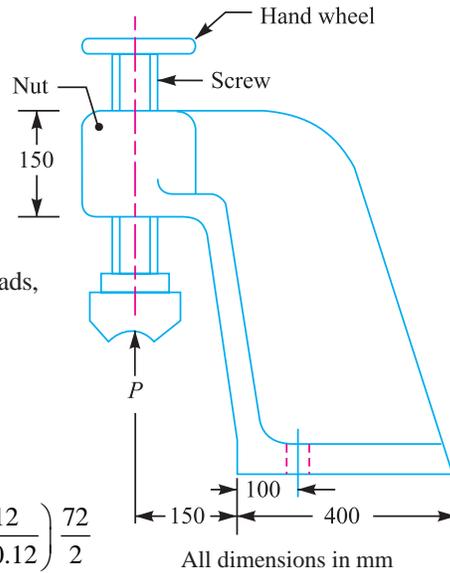


Fig. 17.7

We know that the torque required at the rim of handwheel (T),

$$158\,728 = P_1 \times \frac{D}{2} = P_1 \times \frac{300}{2} = 150 P_1$$

$$\therefore P_1 = 158\,728 / 150 = 1058 \text{ N Ans.}$$

2. Maximum compressive stress in the screw

We know that maximum compressive stress in the screw,

$$\sigma_c = \frac{W}{A_c} = \frac{W}{\frac{\pi}{4} (d_c)^2} = \frac{30 \times 10^3}{\frac{\pi}{4} (69)^2} = 8.02 \text{ N/mm}^2 = 8.02 \text{ MPa Ans.}$$

Bearing pressure on the threads

We know that number of threads in contact with the nut,

$$n = \frac{\text{Height of nut}}{\text{Pitch of threads}} = \frac{150}{6} = 25 \text{ threads}$$

and thickness of threads, $t = p/2 = 6/2 = 3 \text{ mm}$

We know that bearing pressure on the threads,

$$p_b = \frac{W}{\pi d \cdot t \cdot n} = \frac{30 \times 10^3}{\pi \times 72 \times 3 \times 25} = 1.77 \text{ N/mm}^2 \text{ Ans.}$$

Maximum shear stress in the threads

We know that shear stress in the threads,

$$\tau = \frac{16 T}{\pi (d_c)^3} = \frac{16 \times 158\,728}{\pi (69)^3} = 2.46 \text{ N/mm}^2 \text{ Ans.}$$

* The mean diameter of the screw (d) is also given by

$$d = d_o - p/2 = 75 - 6/2 = 72 \text{ mm}$$

∴ Maximum shear stress in the threads,

$$\begin{aligned} \tau_{max} &= \frac{1}{2} \sqrt{(\sigma_c)^2 + 4 \tau^2} = \frac{1}{2} \sqrt{(8.02)^2 + 4 (2.46)^2} \\ &= 4.7 \text{ N/mm}^2 = 4.7 \text{ MPa} \quad \text{Ans.} \end{aligned}$$

3. Efficiency of the straightener

We know that the torque required with no friction,

$$T_0 = W \tan \alpha \times \frac{d}{2} = 30 \times 10^3 \times 0.0265 \times \frac{72}{2} = 28\,620 \text{ N-mm}$$

∴ Efficiency of the straightener,

$$\eta = \frac{T_0}{T} = \frac{28\,620}{158\,728} = 0.18 \text{ or } 18\% \quad \text{Ans.}$$

Example 17.9. A sluice gate weighing 18 kN is raised and lowered by means of square threaded screws, as shown in Fig. 17.8. The frictional resistance induced by water pressure against the gate when it is in its lowest position is 4000 N.

The outside diameter of the screw is 60 mm and pitch is 10 mm. The outside and inside diameter of washer is 150 mm and 50 mm respectively. The coefficient of friction between the screw and nut is 0.1 and for the washer and seat is 0.12. Find :

1. The maximum force to be exerted at the ends of the lever raising and lowering the gate,
2. Efficiency of the arrangement, and
3. Number of threads and height of nut, for an allowable bearing pressure of 7 N/mm².

Solution. Given : $W_1 = 18 \text{ kN} = 18\,000 \text{ N}$; $F = 4000 \text{ N}$; $d_o = 60 \text{ mm}$; $p = 10 \text{ mm}$; $D_1 = 150 \text{ mm}$ or $R_1 = 75 \text{ mm}$; $D_2 = 50 \text{ mm}$ or $R_2 = 25 \text{ mm}$; $\mu = \tan \phi = 0.1$; $\mu_1 = 0.12$; $p_b = 7 \text{ N/mm}^2$

1. Maximum force to be exerted at the ends of lever

Let $P_1 =$ Maximum force exerted at each end of the lever 1 m (1000 mm) long.

We know that inner diameter or core diameter of the screw,

$$d_c = d_o - p = 60 - 10 = 50 \text{ mm}$$

Mean diameter of the screw,

$$d = \frac{d_o + d_c}{2} = \frac{60 + 50}{2} = 55 \text{ mm}$$

and $\tan \alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 55} = 0.058$

(a) For raising the gate

Since the frictional resistance acts in the opposite direction to the motion of screw, therefore for raising the gate, the frictional resistance (F) will act downwards.

∴ Total load acting on the screw,

$$W = W_1 + F = 18\,000 + 4000 = 22\,000 \text{ N}$$

and torque required to overcome friction at the screw,

$$T_1 = P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2} = W \left(\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right) \frac{d}{2}$$

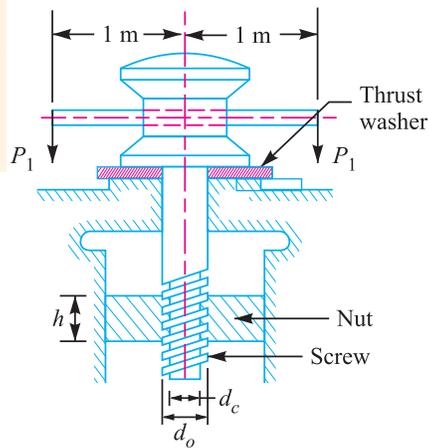


Fig. 17.8

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$$= 22\,000 \left(\frac{0.058 + 0.1}{1 - 0.058 \times 0.1} \right) \frac{55}{2} = 96\,148 \text{ N-mm}$$

Mean radius of washer,

$$R = \frac{R_1 + R_2}{2} = \frac{75 + 25}{2} = 50 \text{ mm}$$

∴ Torque required to overcome friction at the washer,

$$T_2 = \mu_1 WR = 0.12 \times 22\,000 \times 50 = 132\,000 \text{ N-mm}$$

and total torque required to overcome friction,

$$T = T_1 + T_2 = 96\,148 + 132\,000 = 228\,148 \text{ N-mm}$$

We know that the torque required at the end of lever (T),

$$228\,148 = 2P_1 \times \text{Length of lever} = 2P_1 \times 1000 = 2000 P_1$$

∴ $P_1 = 228\,148 / 2000 = 141.1 \text{ N}$ **Ans.**

(b) For lowering the gate

Since the gate is being lowered, therefore the frictional resistance (F) will act upwards,

∴ Total load acting on the screw,

$$W = W_1 - F = 18\,000 - 4000 = 14\,000 \text{ N}$$

We know that torque required to overcome friction at the screw,

$$\begin{aligned} T_1 &= P \times \frac{d}{2} = W \tan(\phi - \alpha) \frac{d}{2} = W \left(\frac{\tan \phi - \tan \alpha}{1 + \tan \phi \tan \alpha} \right) \frac{d}{2} \\ &= 14\,000 \left(\frac{0.1 - 0.058}{1 + 0.1 \times 0.058} \right) \frac{55}{2} = 16\,077 \text{ N-mm} \end{aligned}$$

and torque required to overcome friction at the washer,

$$T_2 = \mu_1 WR = 0.12 \times 14\,000 \times 50 = 84\,000 \text{ N-mm}$$

∴ Total torque required to overcome friction,

$$T = T_1 + T_2 = 16\,077 + 84\,000 = 100\,077 \text{ N-mm}$$

We know that the torque required at the end of lever (T),

$$100\,077 = 2P_1 \times 1000 = 2000 P_1 \quad \text{or} \quad P_1 = 100\,077/2000 = 50.04 \text{ N} \quad \text{Ans.}$$

2. Efficiency of the arrangement

We know that the torque required for raising the load, with no friction,

$$T_0 = W \tan \alpha \times \frac{d}{2} = 22\,000 \times 0.058 \times \frac{55}{2} = 35\,090 \text{ N-mm}$$

∴ Efficiency of the arrangement,

$$\eta = \frac{T_0}{T} = \frac{35\,090}{228\,148} = 0.154 \quad \text{or} \quad 15.4\% \quad \text{Ans.}$$

3. Number of threads and height of nut

Let n = Number of threads in contact with the nut,

h = Height of nut = $n \times p$, and

t = Thickness of thread = $p / 2 = 10 / 2 = 5 \text{ mm}$.

We know that the bearing pressure (p_b),

$$7 = \frac{W}{\pi \cdot d \cdot t \cdot n} = \frac{22000}{\pi \times 55 \times 5 \times n} = \frac{25.46}{n}$$

∴ $n = 25.46 / 7 = 3.64$ say 4 threads **Ans.**

and

$$h = n \times p = 4 \times 10 = 40 \text{ mm} \quad \text{Ans.}$$

Example 17.10. The screw, as shown in Fig. 17.9 is operated by a torque applied to the lower end. The nut is loaded and prevented from turning by guides. Assume friction in the ball bearing to be negligible. The screw is a triple start trapezoidal thread. The outside diameter of the screw is 48 mm and pitch is 8 mm. The coefficient of friction of the threads is 0.15. Find :

1. Load which can be raised by a torque of 40 N-m ;
2. Whether the screw is overhauling ; and
3. Average bearing pressure between the screw and nut thread surface.

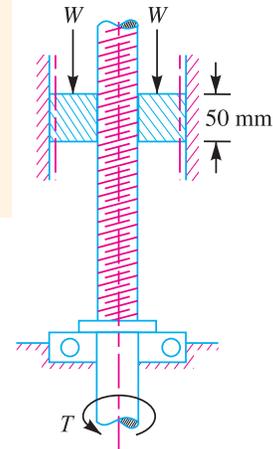


Fig. 17.9

Solution. Given : $d_o = 48$ mm ; $p = 8$ mm ; $\mu = \tan \phi = 0.15$;
 $T = 40$ N-m = 40 000 N-mm

1. Load which can be raised

Let $W =$ Load which can be raised.

We know that mean diameter of the screw,

$$d = d_o - p / 2 = 48 - 8 / 2 = 44 \text{ mm}$$

Since the screw is a triple start, therefore lead of the screw

$$= 3p = 3 \times 8 = 24 \text{ mm}$$

$$\therefore \tan \alpha = \frac{\text{Lead}}{\pi d} = \frac{24}{\pi \times 44} = 0.174$$

and virtual coefficient of friction,

$$\mu_1 = \tan \phi_1 = \frac{\mu}{\cos \beta} = \frac{0.15}{\cos 15^\circ} = \frac{0.15}{0.9659} = 0.155$$

... (\because For trapezoidal threads, $2\beta = 30^\circ$)

We know that the torque required to raise the load,

$$T = P \times \frac{d}{2} = W \tan (\alpha + \phi_1) \frac{d}{2} = W \left[\frac{\tan \alpha + \tan \phi_1}{1 - \tan \alpha \tan \phi_1} \right] \frac{d}{2}$$

$$40\,000 = W \left(\frac{0.174 + 0.155}{1 - 0.174 \times 0.155} \right) \frac{44}{2} = 7.436 W$$

$$\therefore W = 40\,000 / 7.436 = 5380 \text{ N Ans.}$$

2. Whether the screw is overhauling

We know that torque required to lower the load,

$$T = W \tan (\phi_1 - \alpha) \frac{d}{2}$$

We have discussed in Art. 17.9 that if ϕ_1 is less than α , then the torque required to lower the load will be **negative**, i.e. the load will start moving downward without the application of any torque. Such a condition is known as overhauling of screws.

In the present case, $\tan \phi_1 = 0.155$ and $\tan \alpha = 0.174$. Since ϕ_1 is less than α , therefore the screw is overhauling. **Ans.**

3. Average bearing pressure between the screw and nut thread surfaces

We know that height of the nut,

$$h = n \times p = 50 \text{ mm} \quad \dots(\text{Given})$$

\therefore Number of threads in contact,

$$n = h / p = 50 / 8 = 6.25$$

and thickness of thread, $t = p / 2 = 8 / 2 = 4$ mm

We know that the average bearing pressure,

$$p_b = \frac{W}{\pi d t n} = \frac{5380}{\pi \times 44 \times 4 \times 6.25} = 1.56 \text{ N/mm}^2 \text{ Ans.}$$

Example 17.11. A C-clamp, as shown in Fig. 17.10, has trapezoidal threads of 12 mm outside diameter and 2 mm pitch. The coefficient of friction for screw threads is 0.12 and for the collar is 0.25. The mean radius of the collar is 6 mm. If the force exerted by the operator at the end of the handle is 80 N, find: 1. The length of handle; 2. The maximum shear stress in the body of the screw and where does this exist; and 3. The bearing pressure on the threads.

Solution. Given : $d_o = 12 \text{ mm}$; $p = 2 \text{ mm}$; $\mu = \tan \phi = 0.12$; $\mu_2 = 0.25$; $R = 6 \text{ mm}$; $P_1 = 80 \text{ N}$; $W = 4 \text{ kN} = 4000 \text{ N}$

1. Length of handle

Let $l =$ Length of handle.

We know that the mean diameter of the screw,

$$d = d_o - p / 2 = 12 - 2 / 2 = 11 \text{ mm}$$

$$\therefore \tan \alpha = \frac{p}{\pi d} = \frac{2}{\pi \times 11} = 0.058$$

Since the angle for trapezoidal threads is $2\beta = 30^\circ$ or $\beta = 15^\circ$, therefore virtual coefficient of friction,

$$\mu_1 = \tan \phi_1 = \frac{\mu}{\cos \beta} = \frac{0.12}{\cos 15^\circ} = \frac{0.12}{0.9659} = 0.124$$

We know that the torque required to overcome friction at the screw,

$$\begin{aligned} T_1 &= P \times \frac{d}{2} = W \tan (\alpha + \phi_1) \frac{d}{2} = W \left(\frac{\tan \alpha + \tan \phi_1}{1 - \tan \alpha \tan \phi_1} \right) \frac{d}{2} \\ &= 4000 \left(\frac{0.058 + 0.124}{1 - 0.058 \times 0.124} \right) \frac{11}{2} = 4033 \text{ N-mm} \end{aligned}$$

Assuming uniform wear, the torque required to overcome friction at the collar,

$$T_2 = \mu_2 WR = 0.25 \times 4000 \times 6 = 6000 \text{ N-mm}$$

\therefore Total torque required at the end of handle,

$$T = T_1 + T_2 = 4033 + 6000 = 10\,033 \text{ N-mm}$$

We know that the torque required at the end of handle (T),

$$10\,033 = P_1 \times l = 80 \times l \text{ or } l = 10\,033 / 80 = 125.4 \text{ mm Ans.}$$

2. Maximum shear stress in the body of the screw

Consider two sections A-A and B-B. The section A-A just above the nut, is subjected to torque and bending. The section B-B just below the nut is subjected to collar friction torque and direct compressive load. Thus, both the sections must be checked for maximum shear stress.

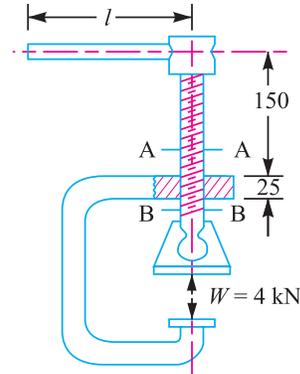
Considering section A-A

We know that the core diameter of the screw,

$$d_c = d_o - p = 12 - 2 = 10 \text{ mm}$$

and torque transmitted at A-A,

$$T = \frac{\pi}{16} \times \tau (d_c)^3$$



All dimensions in mm.

Fig. 17.10

$$\therefore \text{Shear stress, } \tau = \frac{16 T}{\pi (d_c)^3} = \frac{16 \times 10033}{\pi \times 10^3} = 51.1 \text{ N/mm}^2$$

Bending moment at A-A,

$$\begin{aligned} M &= P_1 \times 150 = 80 \times 150 = 12\,000 \text{ N-mm} \\ &= \frac{\pi}{32} \times \sigma_b (d_c)^3 \end{aligned}$$

$$\therefore \text{Bending stress, } \sigma_b = \frac{32 M}{\pi (d_c)^3} = \frac{32 \times 12\,000}{\pi (10)^3} = 122.2 \text{ N/mm}^2$$

We know that the maximum shear stress,

$$\begin{aligned} \tau_{max} &= \frac{1}{2} \sqrt{(\sigma_b)^2 + 4 \tau^2} = \frac{1}{2} \sqrt{(122.2)^2 + 4(51.1)^2} = 79.65 \text{ N/mm}^2 \\ &= 79.65 \text{ MPa} \end{aligned}$$

Considering section B-B

Since the section B-B is subjected to collar friction torque (T_2), therefore the shear stress,

$$\tau = \frac{16 T_2}{\pi (d_c)^3} = \frac{16 \times 6000}{\pi \times 10^3} = 30.6 \text{ N/mm}^2$$

and direct compressive stress,

$$\sigma_c = \frac{W}{A_c} = \frac{4W}{\pi (d_c)^2} = \frac{4 \times 4000}{\pi \times 10^2} = 51 \text{ N/mm}^2$$

\(\therefore\) Maximum shear stress,

$$\tau_{max} = \frac{1}{2} \sqrt{(\sigma_c)^2 + 4 \tau^2} = \frac{1}{2} \sqrt{(51)^2 + 4(30.6)^2} = 39.83 \text{ N/mm}^2 = 39.83 \text{ MPa}$$

From above, we see that the maximum shear stress is 79.65 MPa and occurs at section A-A. **Ans.**

3. Bearing pressure on the threads

We know that height of the nut,

$$h = n \times p = 25 \text{ mm} \quad \dots(\text{Given})$$

\(\therefore\) Number of threads in contact,

$$n = h / p = 25 / 2 = 12.5$$

and thickness of threads, $t = p / 2 = 2 / 2 = 1 \text{ mm}$

We know that bearing pressure on the threads,

$$p_b = \frac{W}{\pi d t n} = \frac{4000}{\pi \times 11 \times 1 \times 12.5} = 9.26 \text{ N/mm}^2 \quad \text{Ans.}$$

Example 17.12. A power transmission screw of a screw press is required to transmit maximum load of 100 kN and rotates at 60 r.p.m. Trapezoidal threads are as under :

Nominal dia, mm	40	50	60	70
Core dia, mm	32.5	41.5	50.5	59.5
Mean dia, mm	36.5	46	55.5	65
Core area, mm ²	830	1353	2003	2781
Pitch, mm	7	8	9	10

The screw thread friction coefficient is 0.12. Torque required for collar friction and journal bearing is about 10% of the torque to drive the load considering screw friction. Determine screw dimensions and its efficiency. Also determine motor power required to drive the screw. Maximum permissible compressive stress in screw is 100 MPa.

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Solution. Given : $W = 100 \text{ kN} = 100 \times 10^3 \text{ N}$;
 $N = 60 \text{ r.p.m.}$; $\mu = 0.12$; $\sigma_c = 100 \text{ MPa} = 100 \text{ N/mm}^2$

Dimensions of the screw

Let $A_c =$ Core area of threads.

We know that the direct compressive stress (σ_c),

$$100 = \frac{W}{A_c} = \frac{100 \times 10^3}{A_c}$$

or $A_c = 100 \times 10^3 / 100 = 1000 \text{ mm}^2$

Since the core area is 1000 mm^2 , therefore we shall use the following dimensions for the screw (for core area 1353 mm^2).

Nominal diameter, $d_o = 50 \text{ mm}$;

Core diameter, $d_c = 41.5 \text{ mm}$;

Mean diameter, $d = 46 \text{ mm}$;

Pitch, $p = 8 \text{ mm}$. **Ans.**

Efficiency of the screw

We know that $\tan \alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 46} = 0.055$

and virtual coefficient of friction,

$$\begin{aligned} \mu_1 &= \tan \phi_1 = \frac{\mu}{\cos \beta} = \frac{0.12}{\cos 15^\circ} \\ &= \frac{0.12}{0.9659} = 0.124 \end{aligned}$$

... (\because For trapezoidal threads, $2\beta = 30^\circ$)

\therefore Force required at the circumference of the screw,

$$\begin{aligned} P &= W \tan (\alpha + \phi_1) = W \left[\frac{\tan \alpha + \tan \phi_1}{1 - \tan \alpha \tan \phi_1} \right] \\ &= 100 \times 10^3 \left[\frac{0.055 + 0.124}{1 - 0.055 \times 0.124} \right] = 18\,023 \text{ N} \end{aligned}$$

and the torque required to drive the load,

$$T_1 = P \times d / 2 = 18\,023 \times 46 / 2 = 414\,530 \text{ N-mm}$$

We know that the torque required for collar friction,

$$T_2 = 10\% T_1 = 0.1 \times 414\,530 = 41\,453 \text{ N-mm}$$

\therefore Total torque required,

$$T = T_1 + T_2 = 414\,530 + 41\,453 = 455\,983 \text{ N-mm} = 455.983 \text{ N-m}$$

We know that the torque required with no friction,

$$T_0 = W \tan \alpha \times \frac{d}{2} = 100 \times 10^3 \times 0.055 \times \frac{46}{2} = 126\,500 \text{ N-mm}$$

\therefore Efficiency of the screw,

$$\eta = \frac{T_0}{T} = \frac{126\,500}{455\,983} = 0.278 \text{ or } 27.8\% \text{ Ans.}$$



This screw press was made in 1735 and installed in the Segovia Mint to strike a new series of copper coins which began in 1772. This press is presently on display in the Alcazar castle of Segovia.

Power required to drive the screw

We know that the power required to drive the screw,

$$= T \times \omega = \frac{T \times 2 \pi N}{60} = \frac{455.683 \times 2 \pi \times 60}{60} = 2865 \text{ W} \\ = 2.865 \text{ kW Ans.}$$

Example 17.13. A vertical two start square threaded screw of 100 mm mean diameter and 20 mm pitch supports a vertical load of 18 kN. The nut of the screw is fitted in the hub of a gear wheel having 80 teeth which meshes with a pinion of 20 teeth. The mechanical efficiency of the pinion and gear wheel drive is 90 percent. The axial thrust on the screw is taken by a collar bearing 250 mm outside diameter and 100 mm inside diameter. Assuming uniform pressure conditions, find, minimum diameter of pinion shaft and height of nut, when coefficient of friction for the vertical screw and nut is 0.15 and that for the collar bearing is 0.20. The permissible shear stress in the shaft material is 56 MPa and allowable bearing pressure is 1.4 N/mm².

Solution. Given : $d = 100 \text{ mm}$; $p = 20 \text{ mm}$; $W = 18 \text{ kN} = 18 \times 10^3 \text{ N}$; No. of teeth on gear wheel = 80 ; No. of teeth on pinion = 20 ; $\eta_m = 90\% = 0.9$; $D_1 = 250 \text{ mm}$ or $R_1 = 125 \text{ mm}$; $D_2 = 100 \text{ mm}$ or $R_2 = 50 \text{ mm}$; $\mu = \tan \phi = 0.15$; $\mu_1 = 0.20$; $\tau = 56 \text{ MPa} = 56 \text{ N/mm}^2$; $p_b = 1.4 \text{ N/mm}^2$

Minimum diameter of pinion shaft

Let $D =$ Minimum diameter of pinion shaft.

Since the screw is a two start square threaded screw, therefore lead of the screw
 $= 2 p = 2 \times 20 = 40 \text{ mm}$

$$\therefore \tan \alpha = \frac{\text{Lead}}{\pi d} = \frac{40}{\pi \times 100} = 0.127$$

and torque required to overcome friction at the screw and nut,

$$T_1 = P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2} = W \left(\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right) \frac{d}{2} \\ = 18 \times 10^3 \left(\frac{0.127 + 0.15}{1 - 0.127 \times 0.15} \right) \frac{100}{2} = 254 \ 160 \text{ N-mm} \\ = 254.16 \text{ N-m}$$

We know that, for uniform pressure conditions, torque required to overcome friction at the collar bearing,

$$T_2 = \frac{2}{3} \times \mu_1 W \left[\frac{(R_1)^3 - (R_2)^3}{(R_1)^2 - (R_2)^2} \right] \\ = \frac{2}{3} \times 0.20 \times 18 \times 10^3 \left[\frac{(125)^3 - (50)^3}{(125)^2 - (50)^2} \right] \text{ N-mm} \\ = 334 \ 290 \text{ N-mm} = 334.29 \text{ N-m}$$

Since the nut of the screw is fixed in the hub of a gear wheel, therefore the total torque required at the gear wheel,

$$T_w = T_1 + T_2 = 254.16 + 334.29 = 588.45 \text{ N-m}$$

Also the gear wheel having 80 teeth meshes with pinion having 20 teeth and the torque is proportional to the number of teeth, therefore torque required at the pinion shaft,

$$= \frac{T_w \times 20}{80} = 588.45 \times \frac{20}{80} = 147.11 \text{ N-m}$$

Since the mechanical efficiency of the pinion and gear wheel is 90%, therefore net torque required at the pinion shaft,

$$T_p = \frac{147.11 \times 100}{90} = 163.46 \text{ N-m} = 163 \ 460 \text{ N-mm}$$

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We know that the torque required at the pinion shaft (T_p),

$$163\,460 = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 56 \times D^3 = 11 D^3$$

$$\therefore D^3 = 163\,460 / 11 = 14\,860 \quad \text{or } D = 24.6 \text{ say } 25 \text{ mm Ans.}$$

Height of nut

Let h = Height of nut,
 n = Number of threads in contact, and
 t = Thickness or width of thread = $p/2 = 20/2 = 10$ mm

We know that the bearing pressure (p_b),

$$1.4 = \frac{W}{\pi d t n} = \frac{18 \times 10^3}{\pi \times 100 \times 10 \times n} = \frac{5.73}{n}$$

$$\therefore n = 5.73 / 1.4 = 4.09 \text{ say } 5 \text{ threads}$$

and height of nut, $h = n \times p = 5 \times 20 = 100$ mm **Ans.**

Example 17.14. A screw press is to exert a force of 40 kN. The unsupported length of the screw is 400 mm. Nominal diameter of screw is 50 mm. The screw has square threads with pitch equal to 10 mm. The material of the screw and nut are medium carbon steel and cast iron respectively. For the steel used take ultimate crushing stress as 320 MPa, yield stress in tension or compression as 200 MPa and that in shear as 120 MPa. Allowable shear stress for cast iron is 20 MPa and allowable bearing pressure between screw and nut is 12 N/mm². Young's modulus for steel = 210 kN/mm². Determine the factor of safety of screw against failure. Find the dimensions of the nut. What is the efficiency of the arrangement? Take coefficient of friction between steel and cast iron as 0.13.

Solution. Given : $W = 40 \text{ kN} = 40 \times 10^3 \text{ N}$; $L = 400 \text{ mm} = 0.4 \text{ m}$; $d_o = 50 \text{ mm}$; $p = 10 \text{ mm}$;
 $\sigma_{cu} = 320 \text{ MPa} = 320 \text{ N/mm}^2$; $\sigma_y = 200 \text{ MPa} = 200 \text{ N/mm}^2$; $\tau_y = 120 \text{ MPa} = 120 \text{ N/mm}^2$;
 $\tau_c = 20 \text{ MPa} = 20 \text{ N/mm}^2$; $p_b = 12 \text{ N/mm}^2$; $E = 210 \text{ kN/mm}^2 = 210 \times 10^3 \text{ N/mm}^2$; $\mu = \tan \phi = 0.13$

We know that the inner diameter or core diameter of the screw,

$$d_c = d_o - p = 50 - 10 = 40 \text{ mm}$$

and core area of the screw,

$$A_c = \frac{\pi}{4} (d_c)^2 = \frac{\pi}{4} (40)^2 = 1257 \text{ mm}^2$$

\therefore Direct compressive stress on the screw due to axial load,

$$\sigma_c = \frac{W}{A_c} = \frac{40 \times 10^3}{1257} = 31.8 \text{ N/mm}^2$$

We know that the mean diameter of the screw,

$$d = \frac{d_o + d_c}{2} = \frac{50 + 40}{2} = 45 \text{ mm}$$

and $\tan \alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 45} = 0.07$

\therefore Torque required to move the screw,

$$\begin{aligned} T &= P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2} = W \left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right] \frac{d}{2} \\ &= 40 \times 10^3 \left[\frac{0.07 + 0.13}{1 - 0.07 \times 0.13} \right] \frac{45}{2} = 181.6 \times 10^3 \text{ N-mm} \end{aligned}$$

We know that torque transmitted by the screw (T),

$$181.6 \times 10^3 = \frac{\pi}{16} \times \tau (d_c)^3 = \frac{\pi}{16} \times \tau (40)^3 = 12\,568 \tau$$

$$\therefore \tau = 181.6 \times 10^3 / 12\,568 = 14.45 \text{ N/mm}^2$$

According to maximum shear stress theory, we have

$$\tau_{max} = \frac{1}{2} \sqrt{(\sigma_c)^2 + 4\tau^2} = \frac{1}{2} \sqrt{(31.8)^2 + 4(14.45)^2} = 21.5 \text{ N/mm}^2$$

Factor of safety

We know that factor of safety

$$= \frac{\tau_y}{\tau_{max}} = \frac{120}{21.5} = 5.58$$

Now considering the screw as a column, assuming one end fixed and other end free. According to J.B. Johnson's formula, critical load,

$$W_{cr} = A_c \times \sigma_y \left[1 - \frac{\sigma_y}{4C \pi^2 E} \left(\frac{L}{k} \right)^2 \right]$$

For one end fixed and other end free, $C = 0.25$.

$$\begin{aligned} \therefore W_{cr} &= 1257 \times 200 \left[1 - \frac{200}{4 \times 0.25 \times \pi^2 \times 210 \times 10^3} \left(\frac{400}{10} \right)^2 \right] \text{ N} \\ &= 212\,700 \text{ N} \end{aligned}$$

... ($\because k = d_c / 4 = 40 / 4 = 10 \text{ mm}$)

$$\therefore \text{Factor of safety} = \frac{W_{cr}}{W} = \frac{212\,700}{40 \times 10^3} = 5.3$$

We shall take larger value of the factor of safety.

$$\therefore \text{Factor of safety} = 5.58 \text{ say } 6 \quad \text{Ans.}$$

Dimensions of the nut

Let n = Number of threads in contact with nut, and

$$h = \text{Height of nut} = p \times n$$

Assume that the load is uniformly distributed over the threads in contact.

We know that the bearing pressure (p_b),

$$12 = \frac{W}{\frac{\pi}{4} [(d_o)^2 - (d_c)^2] n} = \frac{40 \times 10^3}{\frac{\pi}{4} [(50)^2 - (40)^2] n} = \frac{56.6}{n}$$

$$\therefore n = 56.6 / 12 = 4.7 \text{ say } 5 \text{ threads} \quad \text{Ans.}$$

and $h = p \times n = 10 \times 5 = 50 \text{ mm}$ **Ans.**

Now let us check for the shear stress induced in the nut which is of cast iron. We know that

$$\begin{aligned} \tau_{nut} &= \frac{W}{\pi n d_o t} = \frac{40 \times 10^3}{\pi \times 5 \times 50 \times 5} = 10.2 \text{ N/mm}^2 = 10.2 \text{ MPa} \\ &\dots (\because t = p / 2 = 10 / 2 = 5 \text{ mm}) \end{aligned}$$

This value is less than the given value of $\tau_c = 20 \text{ MPa}$, hence the nut is safe.

Efficiency of the arrangement

We know that torque required to move the screw with no friction,

$$T_0 = W \tan \alpha \times \frac{d}{2} = 40 \times 10^3 \times 0.07 \times \frac{45}{2} = 63 \times 10^3 \text{ N-mm}$$

\therefore Efficiency of the arrangement

$$\eta = \frac{T_0}{T} = \frac{63 \times 10^3}{181.6 \times 10^3} = 0.347 \text{ or } 34.7\% \quad \text{Ans.}$$

17.14 Design of Screw Jack

A bottle screw jack for lifting loads is shown in Fig. 17.11. The various parts of the screw jack are as follows:

1. Screwed spindle having square threaded screws,
2. Nut and collar for nut,
3. Head at the top of the screwed spindle for handle,
4. Cup at the top of head for the load, and
5. Body of the screw jack.

In order to design a screw jack for a load W , the following procedure may be adopted:

1. First of all, find the core diameter (d_c) by considering that the screw is under pure compression, *i.e.*

$$W = \sigma_c \times A_c = \sigma_c \times \frac{\pi}{4} (d_c)^2$$

The standard proportions of the square threaded screw are fixed from Table 17.1.

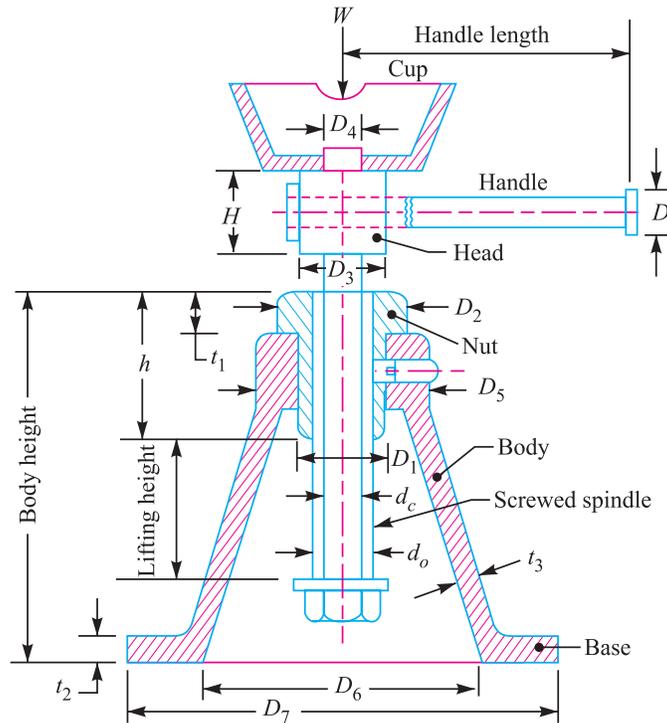


Fig. 17.11. Screw jack.

2. Find the torque (T_1) required to rotate the screw and find the shear stress (τ) due to this torque.

We know that the torque required to lift the load,

$$T_1 = P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2}$$

where

P = Effort required at the circumference of the screw, and

d = Mean diameter of the screw.

∴ Shear stress due to torque T_1 ,

$$\tau = \frac{16 T_1}{\pi (d_c)^3}$$

Also find direct compressive stress (σ_c) due to axial load, *i.e.*

$$\sigma_c = \frac{W}{\frac{\pi}{4} (d_c)^2}$$

3. Find the principal stresses as follows:

Maximum principal stress (tensile or compressive),

$$\sigma_{c(max)} = \frac{1}{2} \left[\sigma_c + \sqrt{(\sigma_c)^2 + 4 \tau^2} \right]$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2} \sqrt{(\sigma_c)^2 + 4 \tau^2}$$

These stresses should be less than the permissible stresses.

4. Find the height of nut (h), considering the bearing pressure on the nut. We know that the bearing pressure on the nut,

$$p_b = \frac{W}{\frac{\pi}{4} [(d_o)^2 - (d_c)^2] n}$$

where

n = Number of threads in contact with screwed spindle.

∴ Height of nut,

$$h = n \times p$$

where

p = Pitch of threads.

5. Check the stresses in the screw and nut as follows :

$$\tau_{(screw)} = \frac{W}{\pi n d_c t}$$

$$\tau_{(nut)} = \frac{W}{\pi n d_o t}$$

where

t = Thickness of screw = $p/2$

6. Find inner diameter (D_1), outer diameter (D_2) and thickness (t_1) of the nut collar.

The inner diameter (D_1) is found by considering the tearing strength of the nut. We know that

$$W = \frac{\pi}{4} [(D_1)^2 - (d_o)^2] \sigma_t$$

The outer diameter (D_2) is found by considering the crushing strength of the nut collar. We know that

$$W = \frac{\pi}{4} [(D_2)^2 - (D_1)^2] \sigma_c$$

The thickness (t_1) of the nut collar is found by considering the shearing strength of the nut collar. We know that

$$W = \pi D_1 t_1 \tau$$

7. Fix the dimensions for the diameter of head (D_3) on the top of the screw and for the cup. Take $D_3 = 1.75 d_o$. The seat for the cup is made equal to the diameter of head and it is chamfered at the top. The cup is fitted with a pin of diameter $D_4 = D_3/4$ approximately. This pin remains a loose fit in the cup.

8. Find the torque required (T_2) to overcome friction at the top of screw. We know that

$$T_2 = \frac{2}{3} \times \mu_1 W \left[\frac{(R_3)^3 - (R_4)^3}{(R_3)^2 - (R_4)^2} \right] \quad \dots \text{(Assuming uniform pressure conditions)}$$

$$= \mu_1 W \left[\frac{R_3 + R_4}{2} \right] = \mu_1 W R \quad \dots \text{(Assuming uniform wear conditions)}$$

where R_3 = Radius of head, and
 R_4 = Radius of pin.

9. Now the total torque to which the handle will be subjected is given by

$$T = T_1 + T_2$$

Assuming that a person can apply a force of 300 – 400 N intermittently, the length of handle required

$$= T / 300$$

The length of handle may be fixed by giving some allowance for gripping.

10. The diameter of handle (D) may be obtained by considering bending effects. We know that bending moment,

$$M = \frac{\pi}{32} \times \sigma_b \times D^3 \quad \dots (\because \sigma_b = \sigma_t \text{ or } \sigma_c)$$

11. The height of head (H) is usually taken as twice the diameter of handle, i.e. $H = 2D$.

12. Now check the screw for buckling load.

Effective length or unsupported length of the screw,

$$L = \text{Lift of screw} + \frac{1}{2} \text{ Height of nut}$$

We know that buckling or critical load,

$$W_{cr} = A_c \cdot \sigma_y \left[1 - \frac{\sigma_y}{4 C \pi^2 E} \left(\frac{L}{k} \right)^2 \right]$$

where σ_y = Yield stress,

C = End fixity coefficient. The screw is considered to be a strut with lower end fixed and load end free. For one end fixed and the other end free, $C = 0.25$

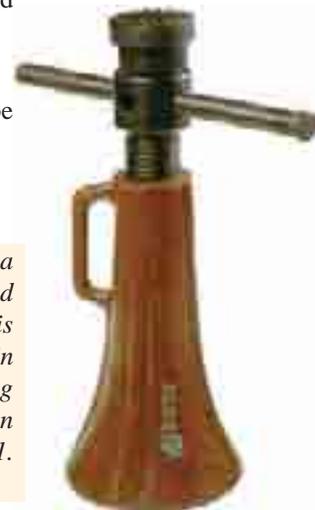
k = Radius of gyration = $0.25 d_c$

The buckling load as obtained by the above expression must be higher than the load at which the screw is designed.

13. Fix the dimensions for the body of the screw jack.

14. Find efficiency of the screw jack.

Example 17.15. A screw jack is to lift a load of 80 kN through a height of 400 mm. The elastic strength of screw material in tension and compression is 200 MPa and in shear 120 MPa. The material for nut is phosphor-bronze for which the elastic limit may be taken as 100 MPa in tension, 90 MPa in compression and 80 MPa in shear. The bearing pressure between the nut and the screw is not to exceed 18 N/mm². Design and draw the screw jack. The design should include the design of 1. screw, 2. nut, 3. handle and cup, and 4. body.



Screw jack

Solution. Given : $W = 80 \text{ kN} = 80 \times 10^3 \text{ N}$; $H_1 = 400 \text{ mm} = 0.4 \text{ m}$; $\sigma_{et} = \sigma_{ec} = 200 \text{ MPa} = 200 \text{ N/mm}^2$; $\tau_e = 120 \text{ MPa} = 120 \text{ N/mm}^2$; $\sigma_{et(nut)} = 100 \text{ MPa} = 100 \text{ N/mm}^2$; $\sigma_{ec(nut)} = 90 \text{ MPa} = 90 \text{ N/mm}^2$; $\tau_{e(nut)} = 80 \text{ MPa} = 80 \text{ N/mm}^2$; $p_b = 18 \text{ N/mm}^2$

The various parts of a screw jack are designed as discussed below:

1. Design of screw for spindle

Let d_c = Core diameter of the screw.

Since the screw is under compression, therefore load (W),

$$80 \times 10^3 = \frac{\pi}{4} (d_c)^2 \times \frac{\sigma_{ec}}{F.S.} = \frac{\pi}{4} (d_c)^2 \frac{200}{2} = 78.55 (d_c)^2$$

... (Taking factor of safety, $F.S. = 2$)

$$\therefore (d_c)^2 = 80 \times 10^3 / 78.55 = 1018.5 \quad \text{or} \quad d_c = 32 \text{ mm}$$

For square threads of normal series, the following dimensions of the screw are selected from Table 17.2.

*Core diameter, $d_c = 38 \text{ mm}$ **Ans.**

Nominal or outside diameter of spindle,

$$d_o = 46 \text{ mm} \text{ **Ans.**}$$

Pitch of threads, $p = 8 \text{ mm}$ **Ans.**

Now let us check for principal stresses:

We know that the mean diameter of screw,

$$d = \frac{d_o + d_c}{2} = \frac{46 + 38}{2} = 42 \text{ mm}$$

and $\tan \alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 42} = 0.0606$

Assuming coefficient of friction between screw and nut,

$$\mu = \tan \phi = 0.14$$

\therefore Torque required to rotate the screw in the nut,

$$\begin{aligned} T_1 &= P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2} = W \left[\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right] \frac{d}{2} \\ &= 80 \times 10^3 \left[\frac{0.0606 + 0.14}{1 - 0.0606 \times 0.14} \right] \frac{42}{2} = 340 \times 10^3 \text{ N-mm} \end{aligned}$$

Now compressive stress due to axial load,

$$\sigma_c = \frac{W}{A_c} = \frac{W}{\frac{\pi}{4} (d_c)^2} = \frac{80 \times 10^3}{\frac{\pi}{4} (38)^2} = 70.53 \text{ N/mm}^2$$

and shear stress due to the torque,

$$\tau = \frac{16 T_1}{\pi (d_c)^3} = \frac{16 \times 340 \times 10^3}{\pi (38)^3} = 31.55 \text{ N/mm}^2$$

\therefore Maximum principal stress (tensile or compressive),

$$\begin{aligned} \sigma_{c(max)} &= \frac{1}{2} \left[\sigma_c + \sqrt{(\sigma_c)^2 + 4 \tau^2} \right] = \frac{1}{2} \left[70.53 + \sqrt{(70.53)^2 + 4 (31.55)^2} \right] \\ &= \frac{1}{2} [70.53 + 94.63] = 82.58 \text{ N/mm}^2 \end{aligned}$$

* From Table 17.2, we see that next higher value of 32 mm for the core diameter is 33 mm. By taking $d_c = 33 \text{ mm}$, gives higher principal stresses than the permissible values. So core diameter is chosen as 38 mm.

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The given value of σ_c is equal to $\frac{\sigma_{ec}}{F.S.}$, i.e. $\frac{200}{2} = 100 \text{ N/mm}^2$

We know that maximum shear stress,

$$\begin{aligned}\tau_{max} &= \frac{1}{2} \left[\sqrt{(\sigma_c)^2 + 4\tau^2} \right] = \frac{1}{2} \left[\sqrt{(70.53)^2 + 4(31.55)^2} \right] \\ &= \frac{1}{2} \times 94.63 = 47.315 \text{ N/mm}^2\end{aligned}$$

The given value of τ is equal to $\frac{\tau_e}{F.S.}$, i.e. $\frac{120}{2} = 60 \text{ N/mm}^2$.

Since these maximum stresses are within limits, therefore design of screw for spindle is safe.

2. Design for nut

Let n = Number of threads in contact with the screwed spindle,
 h = Height of nut = $n \times p$, and
 t = Thickness of screw = $p / 2 = 8 / 2 = 4 \text{ mm}$

Assume that the load is distributed uniformly over the cross-sectional area of nut.

We know that the bearing pressure (p_b),

$$18 = \frac{W}{\frac{\pi}{4} [(d_o)^2 - (d_c)^2] n} = \frac{80 \times 10^3}{\frac{\pi}{4} [(46)^2 - (38)^2] n} = \frac{151.6}{n}$$

$\therefore n = 151.6 / 18 = 8.4$ say 10 threads **Ans.**

and height of nut, $h = n \times p = 10 \times 8 = 80 \text{ mm}$ **Ans.**

Now, let us check the stresses induced in the screw and nut.

We know that shear stress in the screw,

$$\tau_{(screw)} = \frac{W}{\pi n d_c t} = \frac{80 \times 10^3}{\pi \times 10 \times 38 \times 4} = 16.15 \text{ N/mm}^2$$

... ($\because t = p / 2 = 4 \text{ mm}$)

and shear stress in the nut,

$$\tau_{(nut)} = \frac{W}{\pi n d_o t} = \frac{80 \times 10^3}{\pi \times 10 \times 46 \times 4} = 13.84 \text{ N/mm}^2$$

Since these stresses are within permissible limit, therefore design for nut is safe.

Let D_1 = Outer diameter of nut,

D_2 = Outside diameter for nut collar, and

t_1 = Thickness of nut collar.

First of all considering the tearing strength of nut, we have

$$W = \frac{\pi}{4} [(D_1)^2 - (d_o)^2] \sigma_t$$

$$80 \times 10^3 = \frac{\pi}{4} [(D_1)^2 - (46)^2] \frac{100}{2} = 39.3 [(D_1)^2 - 2116] \quad \dots \left[\because \sigma_t = \frac{\sigma_{et(nut)}}{F.S.} \right]$$

or $(D_1)^2 - 2116 = 80 \times 10^3 / 39.3 = 2036$

$\therefore (D_1)^2 = 2036 + 2116 = 4152$ or $D_1 = 65 \text{ mm}$ **Ans.**

Now considering the crushing of the collar of the nut, we have

$$W = \frac{\pi}{4} [(D_2)^2 - (D_1)^2] \sigma_c$$

$$80 \times 10^3 = \frac{\pi}{4} [(D_2)^2 - (65)^2] \frac{90}{2} = 35.3 [(D_2)^2 - 4225] \quad \dots \left[\sigma_c = \frac{\sigma_{ec(nut)}}{F.S.} \right]$$

or $(D_2)^2 - 4225 = 80 \times 10^3 / 35.3 = 2266$

$\therefore (D_2)^2 = 2266 + 4225 = 6491$ or $D_2 = 80.6$ say 82 mm **Ans.**

Considering the shearing of the collar of the nut, we have

$$W = \pi D_1 \times t_1 \times \tau$$

$$80 \times 10^3 = \pi \times 65 \times t_1 \times \frac{80}{2} = 8170 t_1 \quad \dots \left[\tau = \frac{\tau_{e(nut)}}{F.S.} \right]$$

$\therefore t_1 = 80 \times 10^3 / 8170 = 9.8$ say 10 mm **Ans.**

3. Design for handle and cup

The diameter of the head (D_3) on the top of the screwed rod is usually taken as 1.75 times the outside diameter of the screw (d_o).

$\therefore D_3 = 1.75 d_o = 1.75 \times 46 = 80.5$ say 82 mm **Ans.**

The head is provided with two holes at the right angles to receive the handle for rotating the screw. The seat for the cup is made equal to the diameter of head, *i.e.* 82 mm and it is given chamfer at the top. The cup prevents the load from rotating. The cup is fitted to the head with a pin of diameter $D_4 = 20$ mm. The pin remains loose fit in the cup. Other dimensions for the cup may be taken as follows :

Height of cup = 50 mm **Ans.**

Thickness of cup = 10 mm **Ans.**

Diameter at the top of cup = 160 mm **Ans.**

Now let us find out the torque required (T_2) to overcome friction at the top of the screw.

Assuming uniform pressure conditions, we have

$$\begin{aligned} T_2 &= \frac{2}{3} \times \mu_1 W \left[\frac{(R_3)^3 - (R_4)^3}{(R_3)^2 - (R_4)^2} \right] \\ &= \frac{2}{3} \times 0.14 \times 80 \times 10^3 \left[\frac{\left(\frac{82}{2}\right)^3 - \left(\frac{20}{2}\right)^3}{\left(\frac{82}{2}\right)^2 - \left(\frac{20}{2}\right)^2} \right] \quad \dots \text{(Assuming } \mu_1 = \mu) \\ &= 7.47 \times 10^3 \left[\frac{(41)^3 - (10)^3}{(41)^2 - (10)^2} \right] = 321 \times 10^3 \text{ N-mm} \end{aligned}$$

\therefore Total torque to which the handle is subjected,

$$T = T_1 + T_2 = 340 \times 10^3 + 321 \times 10^3 = 661 \times 10^3 \text{ N-mm}$$

Assuming that a force of 300 N is applied by a person intermittently, therefore length of handle required

$$= 661 \times 10^3 / 300 = 2203 \text{ mm}$$

Allowing some length for gripping, we shall take the length of handle as 2250 mm.

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A little consideration will show that an excessive force applied at the end of lever will cause bending. Considering bending effect, the maximum bending moment on the handle,

$$\begin{aligned} M &= \text{Force applied} \times \text{Length of lever} \\ &= 300 \times 2250 = 675 \times 10^3 \text{ N-mm} \end{aligned}$$

Let D = Diameter of the handle.

Assuming that the material of the handle is same as that of screw, therefore taking bending stress $\sigma_b = \sigma_t = \sigma_{et} / 2 = 100 \text{ N/mm}^2$.

We know that the bending moment (M),

$$675 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times D^3 = \frac{\pi}{32} \times 100 \times D^3 = 9.82 D^3$$

$$\therefore D^3 = 675 \times 10^3 / 9.82 = 68.74 \times 10^3 \quad \text{or } D = 40.96 \text{ say } 42 \text{ mm } \mathbf{Ans.}$$

The height of head (H) is taken as $2D$.

$$\therefore H = 2D = 2 \times 42 = 84 \text{ mm } \mathbf{Ans.}$$

Now let us check the screw for buckling load.

We know that the effective length for the buckling of screw,

$$\begin{aligned} L &= \text{Lift of screw} + \frac{1}{2} \text{ Height of nut} = H_1 + h / 2 \\ &= 400 + 80 / 2 = 440 \text{ mm} \end{aligned}$$

When the screw reaches the maximum lift, it can be regarded as a strut whose lower end is fixed and the load end is free. We know that critical load,

$$W_{cr} = A_c \times \sigma_y \left[1 - \frac{\sigma_y}{4C \pi^2 E} \left(\frac{L}{k} \right)^2 \right]$$

For one end fixed and other end free, $C = 0.25$.

$$\text{Also } k = 0.25 d_c = 0.25 \times 38 = 9.5 \text{ mm}$$

$$\begin{aligned} \therefore W_{cr} &= \frac{\pi}{4} (38)^2 200 \left[1 - \frac{200}{4 \times 0.25 \times \pi^2 \times 210 \times 10^3} \left(\frac{440}{9.5} \right)^2 \right] \\ &\dots \text{ (Taking } \sigma_y = \sigma_{et} \text{)} \\ &= 226\,852 (1 - 0.207) = 179\,894 \text{ N} \end{aligned}$$

Since the critical load is more than the load at which the screw is designed (*i.e.* $80 \times 10^3 \text{ N}$), therefore there is no chance of the screw to buckle.

4. Design of body

The various dimensions of the body may be fixed as follows:

Diameter of the body at the top,

$$D_5 = 1.5 D_2 = 1.5 \times 82 = 123 \text{ mm } \mathbf{Ans.}$$

Thickness of the body,

$$t_3 = 0.25 d_o = 0.25 \times 46 = 11.5 \text{ say } 12 \text{ mm } \mathbf{Ans.}$$

Inside diameter at the bottom,

$$D_6 = 2.25 D_2 = 2.25 \times 82 = 185 \text{ mm } \mathbf{Ans.}$$

Outer diameter at the bottom,

$$D_7 = 1.75 D_6 = 1.75 \times 185 = 320 \text{ mm } \mathbf{Ans.}$$

Thickness of base, $t_2 = 2 t_1 = 2 \times 10 = 20 \text{ mm}$ **Ans.**
 Height of the body = Max. lift + Height of nut + 100 mm extra
 = 400 + 80 + 100 = 580 mm **Ans.**

The body is made tapered in order to achieve stability of jack.

Let us now find out the efficiency of the screw jack. We know that the torque required to rotate the screw with no friction,

$$T_0 = W \tan \alpha \times \frac{d}{2} = 80 \times 10^3 \times 0.0606 \times \frac{42}{2} = 101\,808 \text{ N-mm}$$

∴ Efficiency of the screw jack,

$$\eta = \frac{T_0}{T} = \frac{101\,808}{661 \times 10^3} = 0.154 \text{ or } 15.4\% \text{ **Ans.**}$$

Example 17.16. A toggle jack as shown in Fig. 17.12, is to be designed for lifting a load of 4 kN. When the jack is in the top position, the distance between the centre lines of nuts is 50 mm and in the bottom position this distance is 210 mm. The eight links of the jack are symmetrical and 110 mm long. The link pins in the base are set 30 mm apart. The links, screw and pins are made from mild steel for which the permissible stresses are 100 MPa in tension and 50 MPa in shear. The bearing pressure on the pins is limited to 20 N/mm².

Assume the pitch of the square threads as 6 mm and the coefficient of friction between threads as 0.20.

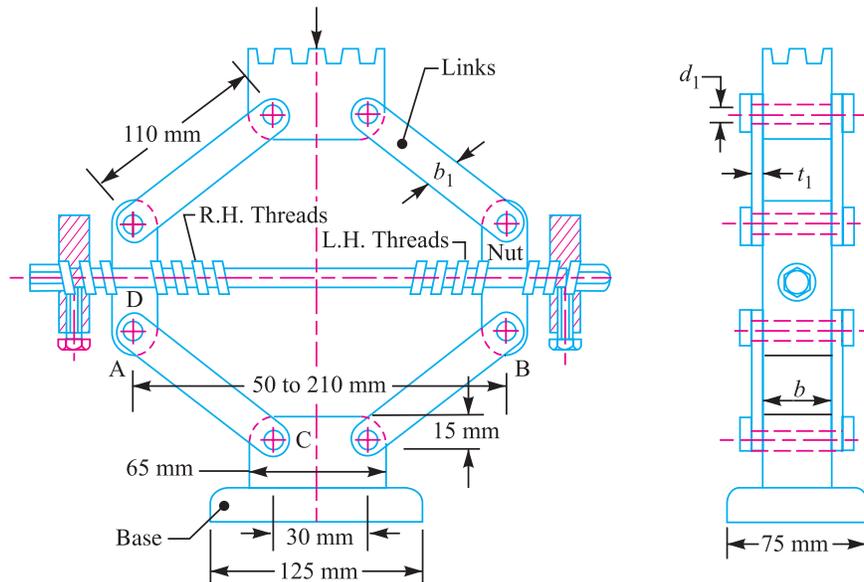


Fig. 17.12

Solution. Given : $W = 4 \text{ kN} = 4000 \text{ N}$; $l = 110 \text{ mm}$; $\sigma_t = 100 \text{ MPa} = 100 \text{ N/mm}^2$; $\tau = 50 \text{ MPa} = 50 \text{ N/mm}^2$; $p_b = 20 \text{ N/mm}^2$; $p = 6 \text{ mm}$; $\mu = \tan \phi = 0.20$

The toggle jack may be designed as discussed below :

1. Design of square threaded screw

A little consideration will show that the maximum load on the square threaded screw occurs when the jack is in the bottom position. The position of the link CD in the bottom position is shown in Fig. 17.13 (a).

Let θ be the angle of inclination of the link CD with the horizontal.

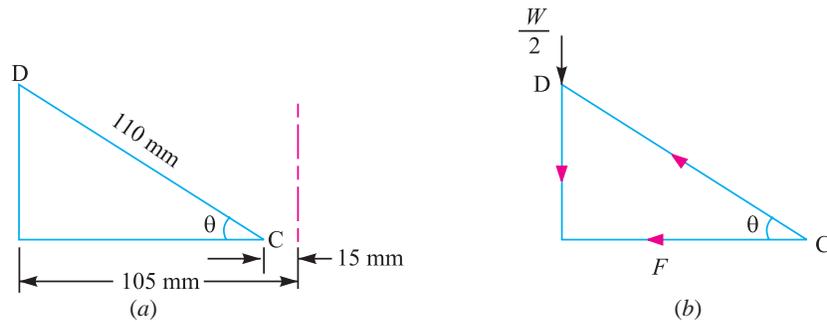


Fig. 17.13

From the geometry of the figure, we find that

$$\cos \theta = \frac{105 - 15}{110} = 0.8112 \text{ or } \theta = 35.1^\circ$$

Each nut carries half the total load on the jack and due to this, the link CD is subjected to tension while the square threaded screw is under pull as shown in Fig. 17.13 (b). The magnitude of the pull on the square threaded screw is given by

$$F = \frac{W}{2 \tan \theta} = \frac{W}{2 \tan 35.1^\circ} = \frac{4000}{2 \times 0.7028} = 2846 \text{ N}$$

Since a similar pull acts on the other nut, therefore total tensile pull on the square threaded rod,

$$W_1 = 2F = 2 \times 2846 = 5692 \text{ N}$$

Let d_c = Core diameter of the screw,

We know that load on the screw (W_1),

$$5692 = \frac{\pi}{4} (d_c)^2 \sigma_t = \frac{\pi}{4} (d_c)^2 100 = 78.55 (d_c)^2$$

$$\therefore (d_c)^2 = 5692 / 78.55 = 72.5 \text{ or } d_c = 8.5 \text{ say } 10 \text{ mm}$$

Since the screw is also subjected to torsional shear stress, therefore to account for this, let us adopt

$$d_c = 14 \text{ mm Ans.}$$

\therefore Nominal or outer diameter of the screw,

$$d_o = d_c + p = 14 + 6 = 20 \text{ mm Ans.}$$

and mean diameter of the screw,

$$d = d_o - p / 2 = 20 - 6 / 2 = 17 \text{ mm}$$

Let us now check for principal stresses. We know that

$$\tan \alpha = \frac{p}{\pi d} = \frac{6}{\pi \times 17} = 0.1123$$

... (where α is the helix angle)

We know that effort required to rotate the screw,

$$P = W_1 \tan (\alpha + \phi) = W_1 \left(\frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi} \right) = 5692 \left(\frac{0.1123 + 0.20}{1 - 0.1123 \times 0.20} \right) = 1822 \text{ N}$$



The rotational speed of the lead screw relative to the spindle speed can be adjusted manually by adding and removing gears to and from the gear

∴ Torque required to rotate the screw,

$$T = P \times \frac{d}{2} = 1822 \times \frac{17}{2} = 15\,487 \text{ N-mm}$$

and shear stress in the screw due to torque,

$$\tau = \frac{16 T}{\pi (d_c)^3} = \frac{16 \times 15\,487}{\pi (14)^3} = 28.7 \text{ N/mm}^2$$

We know that direct tensile stress in the screw,

$$\sigma_t = \frac{W_1}{\frac{\pi}{4} (d_c)^2} = \frac{W_1}{0.7855 (d_c)^2} = \frac{5692}{0.7855 (14)^2} = 37 \text{ N/mm}^2$$

∴ Maximum principal (tensile) stress,

$$\begin{aligned} \sigma_{t(max)} &= \frac{\sigma_t}{2} + \frac{1}{2} \sqrt{(\sigma_t)^2 + 4 \tau^2} = \frac{37}{2} + \frac{1}{2} \sqrt{(37)^2 + 4 (28.7)^2} \\ &= 18.5 + 34.1 = 52.6 \text{ N/mm}^2 \end{aligned}$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2} \sqrt{(\sigma_t)^2 + 4 \tau^2} = \frac{1}{2} \sqrt{(37)^2 + 4 (28.7)^2} = 34.1 \text{ N/mm}^2$$

Since the maximum stresses are within safe limits, therefore the design of square threaded screw is satisfactory.

2. Design of nut

Let n = Number of threads in contact with the screw (*i.e.* square threaded rod).

Assuming that the load W_1 is distributed uniformly over the cross-sectional area of the nut, therefore bearing pressure between the threads (p_b),

$$20 = \frac{W_1}{\frac{\pi}{4} [(d_o)^2 - (d_c)^2] n} = \frac{5692}{\frac{\pi}{4} [(20)^2 - (14)^2] n} = \frac{35.5}{n}$$

$$\therefore n = 35.5 / 20 = 1.776$$

In order to have good stability and also to prevent rocking of the screw in the nut, we shall provide $n = 4$ threads in the nut. The thickness of the nut,

$$t = n \times p = 4 \times 6 = 24 \text{ mm} \quad \text{Ans.}$$

The width of the nut (b) is taken as $1.5 d_o$.

$$\therefore b = 1.5 d_o = 1.5 \times 20 = 30 \text{ mm} \quad \text{Ans.}$$

To control the movement of the nuts beyond 210 mm (the maximum distance between the centre lines of nuts), rings of 8 mm thickness are fitted on the screw with the help of set screws.

$$\begin{aligned} \therefore \text{Length of screwed portion of the screw} \\ &= 210 + t + 2 \times \text{Thickness of rings} \\ &= 210 + 24 + 2 \times 8 = 250 \text{ mm} \quad \text{Ans.} \end{aligned}$$

The central length (about 25 mm) of screwed rod is kept equal to core diameter of the screw *i.e.* 14 mm. Since the toggle jack is operated by means of spanners on both sides of the square threaded rod, therefore the ends of the rod may be reduced to 10 mm square and 15 mm long.

$$\begin{aligned} \therefore \text{Total length of the screw} \\ &= 250 + 2 \times 15 = 280 \text{ mm} \quad \text{Ans.} \end{aligned}$$

Assuming that a force of 150 N is applied by each person at each end of the rod, therefore length of the spanner required

$$= \frac{T}{2 \times 150} = \frac{15487}{300} = 51.62 \text{ mm}$$

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We shall take the length of the spanner as 200 mm in order to facilitate the operation and even a single person can operate it.

3. Design of pins in the nuts

Let d_1 = Diameter of pins in the nuts.

Since the pins are in double shear, therefore load on the pins (F),

$$2846 = 2 \times \frac{\pi}{4} (d_1)^2 \tau = 2 \times \frac{\pi}{4} (d_1)^2 50 = 78.55 (d_1)^2$$

$$\therefore (d_1)^2 = 2846 / 78.55 = 36.23 \text{ or } d_1 = 6.02 \text{ say } 8 \text{ mm Ans.}$$

The diameter of pin head is taken as $1.5 d_1$ (*i.e.* 12 mm) and thickness 4 mm. The pins in the nuts are kept in position by separate rings 4 mm thick and 1.5 mm split pins passing through the rings and pins.

4. Design of links

Due to the load, the links may buckle in two planes at right angles to each other. For buckling in the vertical plane (*i.e.* in the plane of the links), the links are considered as hinged at both ends and for buckling in a plane perpendicular to the vertical plane, it is considered as fixed at both ends. We know that load on the link

$$= F / 2 = 2846 / 2 = 1423 \text{ N}$$

Assuming a factor of safety = 5, the links must be designed for a buckling load of

$$W_{cr} = 1423 \times 5 = 7115 \text{ N}$$

Let t_1 = Thickness of the link, and

b_1 = Width of the link.

Assuming that the width of the link is three times the thickness of the link, *i.e.* $b_1 = 3 t_1$, therefore cross-sectional area of the link,

$$A = t_1 \times 3t_1 = 3(t_1)^2$$

and moment of inertia of the cross-section of the link,

$$I = \frac{1}{12} \times t_1 (b_1)^3 = \frac{1}{12} \times t_1 (3t_1)^3 = 2.25 (t_1)^4$$

We know that the radius of gyration,

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.25 (t_1)^4}{3(t_1)^2}} = 0.866 t_1$$

Since for buckling of the link in the vertical plane, the ends are considered as hinged, therefore equivalent length of the link,

$$L = l = 110 \text{ mm}$$

and Rankine's constant, $a = \frac{1}{7500}$

According to Rankine's formula, buckling load (W_{cr}),

$$7115 = \frac{\sigma_c \times A}{1 + a \left(\frac{L}{k}\right)^2} = \frac{100 \times 3(t_1)^2}{1 + \frac{1}{7500} \left(\frac{110}{0.866 t_1}\right)^2} = \frac{300 (t_1)^2}{1 + \frac{2.15}{(t_1)^2}}$$

or
$$\frac{7115}{300} = \frac{(t_1)^4}{(t_1)^2 + 2.15}$$

$$(t_1)^4 - 23.7 (t_1)^2 - 51 = 0$$

$\therefore (t_1)^2 = \frac{23.7 \pm \sqrt{(23.7)^2 + 4 \times 51}}{2} = \frac{23.7 + 27.7}{2} = 25.7$

or $t_1 = 5.07$ say 6 mm ... (Taking + ve sign)

and $b_1 = 3 t_1 = 3 \times 6 = 18$ mm

Now let us consider the buckling of the link in a plane perpendicular to the vertical plane.

Moment of inertia of the cross-section of the link,

$$I = \frac{1}{12} \times b_1 (t_1)^3 = \frac{1}{12} \times 3 t_1 (t_1)^3 = 0.25 (t_1)^4$$

and cross-sectional area of the link,

$$A = t_1 \cdot b_1 = t_1 \times 3 t_1 = 3 (t_1)^2$$

\therefore Radius of gyration,

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.25 (t_1)^4}{3 (t_1)^2}} = 0.29 t_1$$

Since for buckling of the link in a plane perpendicular to the vertical plane, the ends are considered as fixed, therefore

Equivalent length of the link,

$$L = l / 2 = 110 / 2 = 55 \text{ mm}$$

Again according to Rankine's formula, buckling load,

$$W_{cr} = \frac{\sigma_c \times A}{1 + a \left(\frac{L}{k}\right)^2} = \frac{100 \times 3(t_1)^2}{1 + \frac{1}{7500} \left(\frac{55}{0.29 t_1}\right)^2} = \frac{300(t_1)^2}{1 + \frac{4.8}{(t_1)^2}}$$

Substituting the value of $t_1 = 6$ mm, we have

$$W_{cr} = \frac{300 \times 6^2}{1 + \frac{4.8}{6^2}} = 9532 \text{ N}$$

Since this buckling load is more than the calculated value (*i.e.* 7115 N), therefore the link is safe for buckling in a plane perpendicular to the vertical plane.

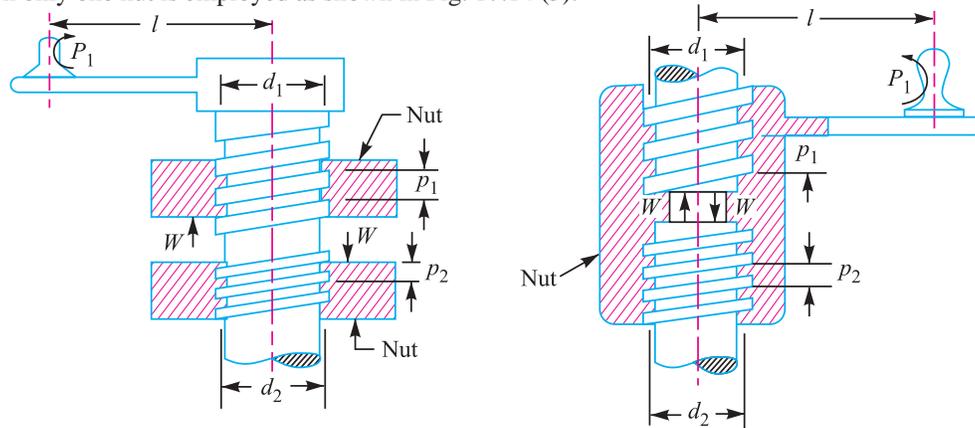
\therefore We may take $t_1 = 6$ mm ; and $b_1 = 18$ mm **Ans.**

17.15 Differential and Compound Screws

There are certain cases in which a very slow movement of the screw is required whereas in other cases, a very fast movement of the screw is needed. The slow movement of the screw may be obtained by using a small pitch of the threads, but it results in weak threads. The fast movement of the screw may be obtained by using multiple-start threads, but this method requires expensive machining and the loss of self-locking property. In order to overcome these difficulties, differential or compound screws, as discussed below, are used.

1. Differential screw. When a slow movement or fine adjustment is desired in precision equipments, then a differential screw is used. It consists of two threads of the same hand (*i.e.* right handed or left handed) but of different pitches, wound on the same cylinder or different cylinders as shown in Fig. 17.14. It may be noted that when the threads are wound on the same cylinder, then two

nuts are employed as shown in Fig. 17.14 (a) and when the threads are wound on different cylinders, then only one nut is employed as shown in Fig. 17.14 (b).



(a) Threads wound on the same cylinder.

(b) Threads wound on the different cylinders.

Fig. 17.14

In this case, each revolution of the screw causes the nuts to move towards or away from each other by a distance equal to the difference of the pitches.

Let p_1 = Pitch of the upper screw,
 d_1 = Mean diameter of the upper screw,
 α_1 = Helix angle of the upper screw, and
 μ_1 = Coefficient of friction between the upper screw and the upper nut
 = $\tan \phi_1$, where ϕ_1 is the friction angle.

p_2, d_2, α_2 and μ_2 = Corresponding values for the lower screw.

We know that torque required to overcome friction at the upper screw,

$$T_1 = W \tan (\alpha_1 + \phi_1) \frac{d_1}{2} = W \left[\frac{\tan \alpha_1 + \tan \phi_1}{1 - \tan \alpha_1 \tan \phi_1} \right] \frac{d_1}{2} \quad \dots(i)$$

Similarly, torque required to overcome friction at the lower screw,

$$T_2 = W \tan (\alpha_2 + \phi_2) \frac{d_2}{2} = W \left[\frac{\tan \alpha_2 + \tan \phi_2}{1 - \tan \alpha_2 \tan \phi_2} \right] \frac{d_2}{2} \quad \dots(ii)$$

∴ Total torque required to overcome friction at the thread surfaces,

$$T = P_1 \times l = T_1 - T_2$$

When there is no friction between the thread surfaces, then $\mu_1 = \tan \phi_1 = 0$ and $\mu_2 = \tan \phi_2 = 0$. Substituting these values in the above expressions, we have

$$\therefore T_1' = W \tan \alpha_1 \times \frac{d_1}{2}$$

and $T_2' = W \tan \alpha_2 \times \frac{d_2}{2}$

∴ Total torque required when there is no friction,

$$\begin{aligned} T_0 &= T_1' - T_2' \\ &= W \tan \alpha_1 \times \frac{d_1}{2} - W \tan \alpha_2 \times \frac{d_2}{2} \end{aligned}$$

$$= W \left[\frac{p_1}{\pi d_1} \times \frac{d_1}{2} - \frac{p_2}{\pi d_2} \times \frac{d_2}{2} \right] = \frac{W}{2\pi} (p_1 - p_2)$$

$$\left[\because \tan \alpha_1 = \frac{p_1}{\pi d_1}; \text{ and } \tan \alpha_2 = \frac{p_2}{\pi d_2} \right]$$

We know that efficiency of the differential screw,

$$\eta = \frac{T_0}{T}$$

2. Compound screw. When a fast movement is desired, then a compound screw is employed. It consists of two threads of opposite hands (*i.e.* one right handed and the other left handed) wound on the same cylinder or different cylinders, as shown in Fig. 17.15 (a) and (b) respectively.

In this case, each revolution of the screw causes the nuts to move towards one another equal to the sum of the pitches of the threads. Usually the pitch of both the threads are made equal.

We know that torque required to overcome friction at the upper screw,

$$T_1 = W \tan (\alpha_1 + \phi_1) \frac{d_1}{2} = W \left[\frac{\tan \alpha_1 + \tan \phi_1}{1 - \tan \alpha_1 \tan \phi_1} \right] \frac{d_1}{2} \quad \dots(i)$$

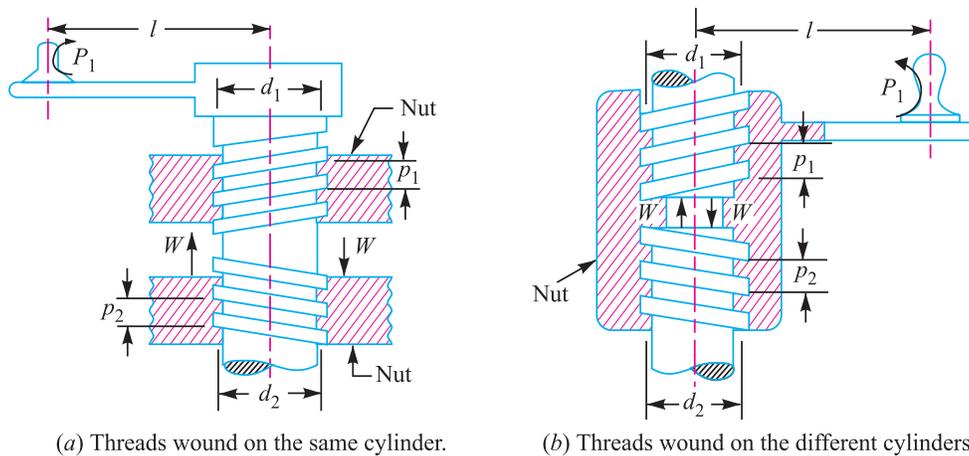


Fig. 17.15

Similarly, torque required to overcome friction at the lower screw,

$$T_2 = W \tan (\alpha_2 + \phi_2) \frac{d_2}{2} = W \left[\frac{\tan \alpha_2 + \tan \phi_2}{1 - \tan \alpha_2 \tan \phi_2} \right] \frac{d_2}{2} \quad \dots(ii)$$

∴ Total torque required to overcome friction at the thread surfaces,

$$T = P_1 \times l = T_1 + T_2$$

When there is no friction between the thread surfaces, then $\mu_1 = \tan \phi_1 = 0$ and $\mu_2 = \tan \phi_2 = 0$. Substituting these values in the above expressions, we have

$$T_1' = W \tan \alpha_1 \times \frac{d_1}{2}$$

$$T_2' = W \tan \alpha_2 \times \frac{d_2}{2}$$

∴ Total torque required when there is no friction,

$$\begin{aligned} T_0 &= T_1' + T_2' \\ &= W \tan \alpha_1 \times \frac{d_2}{2} + W \tan \alpha_2 \times \frac{d_2}{2} \\ &= W \left[\frac{p_1}{\pi d_1} \times \frac{d_1}{2} + \frac{p_2}{\pi d_2} \times \frac{d_2}{2} \right] = \frac{W}{2\pi} (p_1 + p_2) \end{aligned}$$

We know that efficiency of the compound screw,

$$\eta = \frac{T_0}{T}$$

Example 17.17. A differential screw jack is to be made as shown in Fig. 17.16. Neither screw rotates. The outside screw diameter is 50 mm. The screw threads are of square form single start and the coefficient of thread friction is 0.15.

Determine : 1. Efficiency of the screw jack; 2. Load that can be lifted if the shear stress in the body of the screw is limited to 28 MPa.

Solution. Given : $d_o = 50$ mm ; $\mu = \tan \phi = 0.15$; $p_1 = 16$ mm ; $p_2 = 12$ mm ; $\tau_{max} = 28$ MPa = 28 N/mm²

1. Efficiency of the screw jack

We know that the mean diameter of the upper screw,

$$d_1 = d_o - p_1 / 2 = 50 - 16 / 2 = 42 \text{ mm}$$

and mean diameter of the lower screw,

$$d_2 = d_o - p_2 / 2 = 50 - 12 / 2 = 44 \text{ mm}$$

$$\therefore \tan \alpha_1 = \frac{p_1}{\pi d_1} = \frac{16}{\pi \times 42} = 0.1212$$

and
$$\tan \alpha_2 = \frac{p_2}{\pi d_2} = \frac{12}{\pi \times 44} = 0.0868$$

Let W = Load that can be lifted in N.

We know that torque required to overcome friction at the upper screw,

$$\begin{aligned} T_1 &= W \tan (\alpha_1 + \phi) \frac{d_1}{2} = W \left[\frac{\tan \alpha_1 + \tan \phi}{1 - \tan \alpha_1 \tan \phi} \right] \frac{d_1}{2} \\ &= W \left[\frac{0.1212 + 0.15}{1 - 0.1212 \times 0.15} \right] \frac{42}{2} = 5.8 W \text{ N-mm} \end{aligned}$$

Similarly, torque required to overcome friction at the lower screw,

$$\begin{aligned} T_2 &= W \tan (\alpha_2 - \phi) \frac{d_2}{2} = W \left[\frac{\tan \alpha_2 - \tan \phi}{1 + \tan \alpha_2 \tan \phi} \right] \frac{d_2}{2} \\ &= W \left[\frac{0.0868 - 0.15}{1 + 0.0868 \times 0.15} \right] \frac{44}{2} = -1.37 W \text{ N-mm} \end{aligned}$$

∴ Total torque required to overcome friction,

$$T = T_1 - T_2 = 5.8 W - (-1.37 W) = 7.17 W \text{ N-mm}$$

We know that the torque required when there is no friction,

$$T_0 = \frac{W}{2\pi} (p_1 - p_2) = \frac{W}{2\pi} (16 - 12) = 0.636 W \text{ N-mm}$$

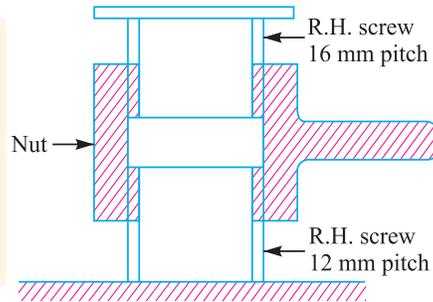


Fig. 17.16. Differential screw.

∴ Efficiency of the screw jack,

$$\eta = \frac{T_0}{T} = \frac{0.636 W}{7.17 W} = 0.0887 \text{ or } 8.87\% \text{ Ans.}$$

2. Load that can be lifted

Since the upper screw is subjected to a larger torque, therefore the load to be lifted (W) will be calculated on the basis of larger torque (T_1).

We know that core diameter of the upper screw,

$$d_{c1} = d_o - p_1 = 50 - 16 = 34 \text{ mm}$$

Since the screw is subjected to direct compressive stress due to load W and shear stress due to torque T_1 , therefore

Direct compressive stress,

$$\sigma_c = \frac{W}{A_{c1}} = \frac{W}{\frac{\pi}{4} (d_{c1})^2} = \frac{W}{\frac{\pi}{4} (34)^2} = \frac{W}{908} \text{ N/mm}^2$$

and shear stress, $\tau = \frac{16 T_1}{\pi (d_{c1})^3} = \frac{16 \times 5.8 W}{\pi (34)^3} = \frac{W}{1331} \text{ N/mm}^2$

We know that maximum shear stress (τ_{max}),

$$\begin{aligned} 28 &= \frac{1}{2} \sqrt{(\sigma_c)^2 + 4 \tau^2} = \frac{1}{2} \sqrt{\left(\frac{W}{908}\right)^2 + 4 \left(\frac{W}{1331}\right)^2} \\ &= \frac{1}{2} \sqrt{1.213 \times 10^{-6} W^2 + 2.258 \times 10^{-6} W^2} = \frac{1}{2} 1.863 \times 10^{-3} W \end{aligned}$$

$$\therefore W = \frac{28 \times 2}{1.863 \times 10^{-3}} = 30\,060 \text{ N} = 30.06 \text{ kN} \text{ Ans.}$$

EXERCISES

1. In a hand vice, the screw has double start square threads of 24 mm outside diameter. If the lever is 200 mm long and the maximum force that can be applied at the end of lever is 250 N, find the force with which the job is held in the jaws of the vice. Assume a coefficient of friction of 0.12. [Ans. 17 420 N]
2. A square threaded bolt of mean diameter 24 mm and pitch 5 mm is tightened by screwing a nut whose mean diameter of bearing surface is 50 mm. If the coefficient of friction for the nut and bolt is 0.1 and for the nut and bearing surfaces 0.16, find the force required at the end of a spanner 0.5 m long when the load on the bolt is 10 kN. [Ans. 120 N]

3. The spindle of a screw jack has a single start square thread with an outside diameter of 45 mm and a pitch of 10 mm. The spindle moves in a fixed nut. The load is carried on a swivel head but is not free to rotate. The bearing surface of the swivel head has a mean diameter of 60 mm. The coefficient of friction between the nut and screw is 0.12 and that between the swivel head and the spindle is 0.10. Calculate the load which can be raised by efforts of 100 N each applied at the end of two levers each of effective length of 350 mm. Also determine the efficiency of the lifting arrangement.

[Ans. 9945 N ; 22.7%]



Lead screw supported by collar bearing.

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4. The cross bar of a planner weighing 12 kN is raised and lowered by means of two square threaded screws of 38 mm outside diameter and 7 mm pitch. The screw is made of steel and a bronze nut of 38 mm thick. A steel collar has 75 mm outside diameter and 38 mm inside diameter. The coefficient of friction at the threads is assumed as 0.11 and at the collar 0.13. Find the force required at a radius of 100 mm to raise and lower the load. [Ans. 402.5 N ; 267 N]
5. The lead screw of a lathe has square threads of 24 mm outside diameter and 5 mm pitch. In order to drive the tool carriage, the screw exerts an axial pressure of 2.5 kN. Find the efficiency of the screw and the power required to drive the screw, if it is to rotate at 30 r.p.m. Neglect bearing friction. Assume coefficient of friction of screw threads as 0.12. [Ans. 37.76% ; 16.55 W]
6. The lead screw of a lathe has Acme threads of 60 mm outside diameter and 8 mm pitch. It supplies drive to a tool carriage which needs an axial force of 2000 N. A collar bearing with inner and outer radius as 30 mm and 60 mm respectively is provided. The coefficient of friction for the screw threads is 0.12 and for the collar it is 0.10. Find the torque required to drive the screw and the efficiency of the screw. [Ans. 18.5 N-m ; 13.6%]
7. A cross bar of a planer weighing 9 kN is raised and lowered by means of two square threaded screws of 40 mm outside diameter and 6 mm pitch. The screw is made of steel and nut of phosphor bronze having 42 mm height. A steel collar bearing with 30 mm mean radius takes the axial thrust. The coefficient of friction at the threads and at the collar may be assumed as 0.14 and 0.10 respectively. Find the force required at a radius of 120 mm of a handwheel to raise and lower the load. Find also the shear stress in the nut material and the bearing pressure on the threads. [Ans. 495 N, 346 N ; 1.7 MPa ; 1.84 N/mm²]
8. A machine slide weighing 3000 N is elevated by a double start acme threaded screw at the rate of 840 mm/min. If the coefficient of friction be 0.12, calculate the power to drive the slide. The end of the screw is carried on a thrust collar of 32 mm inside diameter and 58 mm outside diameter. The pitch of the screw thread is 6 mm and outside diameter of the screw is 40 mm. If the screw is of steel, is it strong enough to sustain the load? Draw a neat sketch of the system. [Ans. 0.165 kW]
9. A sluice valve, used in water pipe lines, consists of a gate raised by the spindle, which is rotated by the hand wheel. The spindle has single start square threads. The nominal diameter of the spindle is 36 mm and the pitch is 6 mm. The friction collar has inner and outer diameters of 32 mm and 50 mm respectively. The coefficient of friction at the threads and the collar are 0.12 and 0.18 respectively. The weight of the gate is 7.5 kN and the frictional resistance to open the valve due to water pressure is 2.75 kN. Using uniform wear theory, determine : 1. torque required to raise the gate; and 2. overall efficiency. [Ans. 136.85 N-m ; 7.1%]
10. A vertical square threads screw of a 70 mm mean diameter and 10 mm pitch supports a vertical load of 50 kN. It passes through the boss of a spur gear wheel of 70 teeth which acts as a nut. In order to raise the load, the spur gear wheel is turned by means of a pinion having 20 teeth. The mechanical efficiency of pinion and gear wheel drive is 90%. The axial thrust on the screw is taken up by a collar bearing having a mean radius of 100 mm. The coefficient of friction for the screw and nut is 0.15 and that for collar bearing is 0.12. Find:
 - (a) Torque to be applied to the pinion shaft,
 - (b) Maximum principal and shear stresses in the screw ; and
 - (c) Height of nut, if the bearing pressure is limited to 12 N/mm².[Ans. 299.6 N-m ; 26.6 N/mm², 19 N/mm² ; 40 mm]
11. A single start square threaded screw is to be designed for a C-clamp. The axial load on the screw may be assumed to be 10 kN. A thrust pad is attached at the end of the screw whose mean diameter may be taken as 30 mm. The coefficient of friction for the screw threads and for the thrust pads is 0.12 and 0.08 respectively. The allowable tensile strength of the screw is 60 MPa and the allowable bearing pressure is 12 N/mm². Design the screw and nut. The square threads are as under :

Nominal diameter, mm	16	18	20	22
Core diameter, mm	13	15	17	19
Pitch, mm	3	3	3	3

[Ans. $d_c = 17$ mm ; $n = 10$, $h = 30$ mm]

12. Design a screw jack for lifting a load of 50 kN through a height of 0.4 m. The screw is made of steel and nut of bronze. Sketch the front sectional view. The following allowable stresses may be assumed
 For steel : Compressive stress = 80 MPa ; Shear stress = 45 MPa
 For bronze : Tensile stress = 40 MPa ; Bearing stress = 15 MPa
 Shear stress = 25 MPa.
- The coefficient of friction between the steel and bronze pair is 0.12. The dimensions of the swivel base may be assumed proportionately. The screw should have square threads. Design the screw, nut and handle. The handle is made of steel having bending stress 150 MPa (allowable).
13. A screw jack carries a load of 22 kN. Assuming the coefficient of friction between screw and nut as 0.15, design the screw and nut. Neglect collar friction and column action. The permissible compressive and shear stresses in the screw should not exceed 42 MPa and 28 MPa respectively. The shear stress in the nut should not exceed 21 MPa. The bearing pressure on the nut is 14 N/mm². Also determine the effort required at the handle of 200 mm length in order to raise and lower the load. What will be the efficiency of screw?
 [Ans. $d_c = 30 \text{ mm}$; $h = 36 \text{ mm}$; 381 N ; 166 N ; 27.6%]
14. Design and draw a screw jack for lifting a safe load of 150 kN through a maximum lift of 350 mm. The elastic strength of the material of the screw may be taken as 240 MPa in compression and 160 MPa in shear. The nut is to be made of phosphor bronze for which the elastic strengths in tension, compression and shear are respectively 130, 115 and 100 MPa. Bearing pressure between the threads of the screw and the nut may be taken as 18 N/mm². Safe crushing stress for the material of the body is 100 MPa. Coefficient of friction for the screw as well as collar may be taken as 0.15.
15. Design a toggle jack to lift a load of 5 kN. The jack is to be so designed that the distance between the centre lines of nuts varies from 50 to 220 mm. The eight links are symmetrical and 120 mm long. The link pins in the base are set 30 mm apart. The links, screw and pins are made from mild steel for which the stresses are 90 MPa in tension and 50 MPa in shear. The bearing pressure on the pin is 20 N/mm². Assume the coefficient of friction between screw and nut as 0.15 and pitch of the square threaded screw as 6 mm.
 [Ans. $d_c = 10 \text{ mm}$; $d_o = 22 \text{ mm}$; $d = 19 \text{ mm}$; $n = 4$; $t = 24 \text{ mm}$; $b = 33 \text{ mm}$; $d_1 = 10 \text{ mm}$; $t_1 = 7 \text{ mm}$; $b_1 = 21 \text{ mm}$]

QUESTIONS

1. Discuss the various types of power threads. Give atleast two practical applications for each type. Discuss their relative advantages and disadvantages.
2. Why are square threads preferable to V-threads for power transmission?
3. How does the helix angle influence on the efficiency of square threaded screw?
4. What do you understand by overhauling of screw?
5. What is self locking property of threads and where it is necessary?
6. Show that the efficiency of self locking screws is less than 50 percent.
7. In the design of power screws, on what factors does the thread bearing pressure depend? Explain.
8. Why is a separate nut preferable to an integral nut with the body of a screw jack?
9. Differentiate between differential screw and compound screw.



Screw jack building-block system

OBJECTIVE TYPE QUESTIONS

1. Which of the following screw thread is adopted for power transmission in either direction?
 - (a) Acme threads
 - (b) Square threads
 - (c) Buttress threads
 - (d) Multiple threads
2. Multiple threads are used to secure
 - (a) low efficiency
 - (b) high efficiency
 - (c) high load lifting capacity
 - (d) high mechanical advantage
3. Screws used for power transmission should have
 - (a) low efficiency
 - (b) high efficiency
 - (c) very fine threads
 - (d) strong teeth
4. If α denotes the lead angle and ϕ , the angle of friction, then the efficiency of the screw is written as
 - (a) $\frac{\tan(\alpha - \phi)}{\tan \alpha}$
 - (b) $\frac{\tan \alpha}{\tan(\alpha - \phi)}$
 - (c) $\frac{\tan(\alpha + \phi)}{\tan \alpha}$
 - (d) $\frac{\tan \alpha}{\tan(\alpha + \phi)}$
5. A screw jack has square threads and the lead angle of the thread is α . The screw jack will be self-locking when the coefficient of friction (μ) is
 - (a) $\mu > \tan \alpha$
 - (b) $\mu = \sin \alpha$
 - (c) $\mu = \cot \alpha$
 - (d) $\mu = \operatorname{cosec} \alpha$
6. To ensure self locking in a screw jack, it is essential that the helix angle is
 - (a) larger than friction angle
 - (b) smaller than friction angle
 - (c) equal to friction angle
 - (d) such as to give maximum efficiency in lifting
7. A screw is said to be self locking screw, if its efficiency is
 - (a) less than 50%
 - (b) more than 50%
 - (c) equal to 50%
 - (d) none of these
8. A screw is said to be over hauling screw, if its efficiency is
 - (a) less than 50%
 - (b) more than 50%
 - (c) equal to 50%
 - (d) none of these
9. While designing a screw in a screw jack against buckling failure, the end conditions for the screw are taken as
 - (a) both ends fixed
 - (b) both ends hinged
 - (c) one end fixed and other end hinged
 - (d) one end fixed and other end free.
10. The load cup of a screw jack is made separate from the head of the spindle to
 - (a) enhance the load carrying capacity of the jack
 - (b) reduce the effort needed for lifting the working load
 - (c) reduce the value of frictional torque required to be countered for lifting the load
 - (d) prevent the rotation of load being lifted

ANSWERS

- | | | | | |
|--------|--------|--------|--------|---------|
| 1. (b) | 2. (b) | 3. (b) | 4. (d) | 5. (a) |
| 6. (b) | 7. (a) | 8. (b) | 9. (d) | 10. (d) |